$\mathcal{NQP}$ Manifolds, Courant Algebroids & Lagrangian Relations

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Outline

Ghosts & Graded Geometry

- BV-BRST Framework
- Graded Manifolds

Generalized Geometry

- Courant Algebroids
- $\mathcal{NQP}$-Manifolds of Degree 2

Correspondences

- Substructures
- Relations
- Wehrheim-Woodward Categories

Outlook

Ghosts & Graded Geometry

BV-BRST: Ghosts & Antighosts

The field-antifield configuration of a *BV-BRST* gauge theory:

$$\big(A, c, A^*, c^*, \mathcal{F}\big)$$

- A = **gauge fields**, c = **ghosts**, (A^*, c^*) = **antifields**, $\mathcal{F}$ = **gauge fixing fermion**.

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$$|\omega| = |\mathcal{M}|\,.$$

Generalized Geometry

Def: A **Courant algebroid** is a vector bundle E equipped with a **pairing**, a **bracket** and an **anchor** $\boldsymbol{\varrho} : E \to TM$

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$$\begin{array}{ccc} \mathcal{M} & \dashrightarrow & T^*[2]E[1] \\ \downarrow & & \downarrow \\ E[1] & \hookrightarrow & (E \oplus E^*)[1] \end{array}$$

$$e \longmapsto e \oplus \langle e, \bullet \rangle$$

Theorem [Roytenberg 2002]: On isomorphism classes, we have:

$$\text{Courant Algebroids} \longleftrightarrow_{bij} \mathcal{NQP}\text{-manifolds of degree 2}$$

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- They satisfy the Courant algebroid axioms iff the *classical master equation* holds $\{S, S\} = 0$.

$$\begin{aligned}\{S, S\} = {} & \left(\left(\boldsymbol{\varrho} \circ \boldsymbol{\varrho}^* \, \mathrm{d}x^j\right) \cdot x^i\right) p_i p_j \\ & + \left(\left[\boldsymbol{\varrho}\left(e_\mu\right), \boldsymbol{\varrho}\left(e_\nu\right)\right] - \boldsymbol{\varrho}\left(\left[\!\left[e_\mu, e_\nu\right]\!\right]\right)\right) \cdot x^i p_i \xi^\mu \xi^\nu \\ & + \frac{1}{12} \left\langle \left[\!\left[\left[\!\left[e_\mu, e_\nu\right]\!\right], e_\rho\right]\!\right] + \left[\!\left[e_\nu, \left[\!\left[e_\mu, e_\rho\right]\!\right]\right]\!\right] - \left[\!\left[e_\mu, \left[\!\left[e_\nu, e_\rho\right]\!\right]\right]\!\right], e_\sigma \right\rangle \xi^\mu \xi^\nu \xi^\rho \xi^\sigma\end{aligned}$$

Correspondences

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Theorem (Generalization of [Grutzmann 2010]): For $\mathcal{M}$ the minimal symplectic realization of E:

$$\text{Dirac Structures of } E \longleftrightarrow_{bij} \text{dg-Lagrangian submanifolds of } M$$

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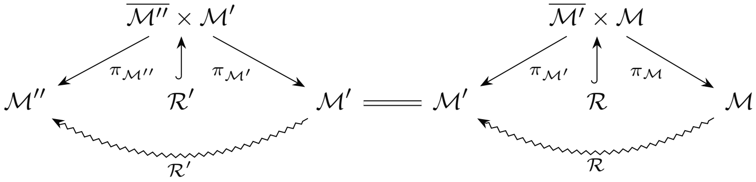
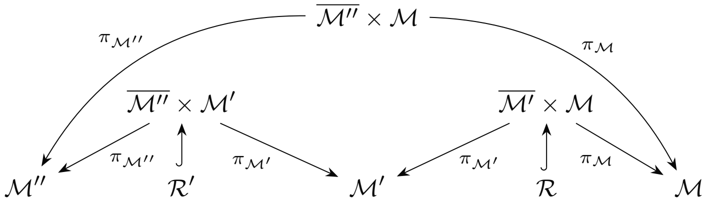
A **dg-Lagrangian relation** $\mathcal{R}$ from $\mathcal{N}$ to $\mathcal{M}$ is a half-dimensional closed submanifold $\mathcal{R} \hookrightarrow \overline{\mathcal{M}} \times \mathcal{N}$ st. the induced sympletic structure $\pi_{\mathcal{N}}^* \omega_{\mathcal{N}} - \pi_{\mathcal{M}}^* \omega_{\mathcal{M}}$ and $\pi_{\mathcal{N}}^* S_{\mathcal{N}} - \pi_{\mathcal{M}}^* S_{\mathcal{M}}$ vanish on $\mathcal{R}$.

Composing Relations

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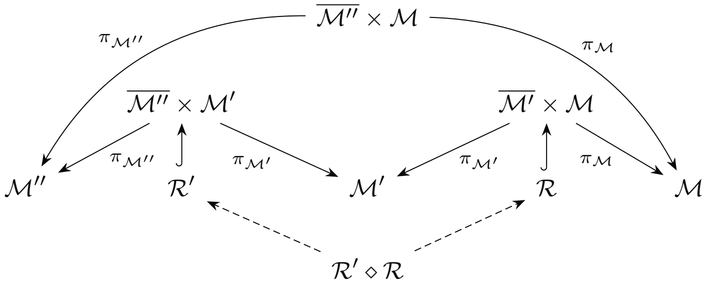
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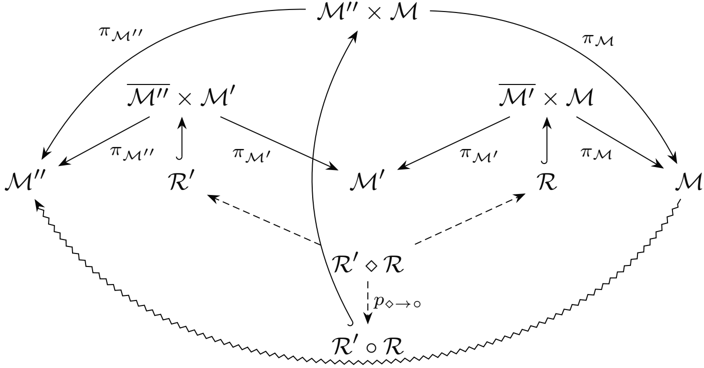
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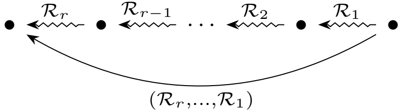

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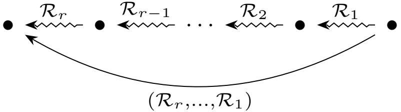

and a corresponding **Wehrheim-Woodward category** [Wehrheim, Woodward 2007]), where composition is given by concatenation and $\big(\mathcal{R}', \mathcal{R}\big) \sim \big(\mathcal{R}' \circ \mathcal{R}\big)$ whenever a composition of relations is defined.

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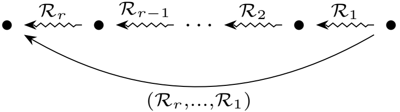

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- Thus we have well-defined categories $\mathcal{CourAlgCorr}$ and $\mathcal{Man}^{\mathbf{2}}_{\mathcal{NQP}}\mathcal{Corr}$.

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$$\mathcal{CourAlgCorr} \simeq \mathcal{Man}^{\mathbf{2}}_{\mathcal{NQP}}\mathcal{Corr}$$

- $\mathcal{OSC} := \mathcal{Man}^{\boldsymbol{Odd}}_{\mathcal{NP}}\mathcal{Corr}$ is the "minimal quantization category" wrt. the odd symplectic quantization functor [Ševera 2006]

Outlook

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THANK YOU FOR YOUR ATTENTION