# Quantum corrections in double field theory

Stanislav Hronek

Masaryk university

Srní 16.1

Based on work with Linus Wulff and Salomon Zacharías
arXiv:2206.10640

# Contents

- Describe $\alpha'$-corrections of "ordinary" supergravity
- Formalism of double field theory
- $\alpha'$-corrections in double field theory
- Outlook and ongoing work

# Underlying theory: "Supergravity"

String theory:

$$x : \Sigma_2 \to M_{10/26}$$

## Worldsheet

- Polyakov action

$$S = -\frac{1}{4\pi\alpha'} \int \mathrm{d}\sigma^2 \sqrt{-g} \partial_i x^m \partial_j x^n \left( g^{ij} G_{mn} + \epsilon^{ij} B_{mn} \right)$$

## Target space effective action

- Background fields: $(G, B, \phi)$.
- Bosonic string low energy effective action ($\frac{\alpha'}{R} \ll 1$):

$$S = \int \mathrm{d}^{26} x \sqrt{-G}\, e^{-2\phi} \left( R - \frac{1}{12} H^2 + \frac{1}{4} \partial^m \phi \partial_m \phi \right).$$

# Supergravity equations of motion

- Also consistency condition for Weyl invariance of string theory at the quantum level.

$$\alpha' R_{mn} + 2\alpha' \nabla_m \nabla_n \phi - \frac{\alpha'}{4} H_{mkl} H_n{}^{kl} = 0$$
$$-\frac{\alpha'}{2} \nabla^k H_{kmn} + \alpha' \nabla^k \phi H_{kmn} = 0$$
$$-\frac{\alpha'}{2} \nabla^2 \phi + \alpha' \nabla_m \phi \nabla^m \phi - \frac{\alpha'}{24} H_{klm} H^{klm} = 0$$

- What are the quantum (higher order in $\alpha'$) corrections?

# Corrections in standard supergravity

$$S = \int \mathrm{d}^{26}x\sqrt{-G}e^{-2\phi}\left(L_0 + \alpha' L_1 + \alpha'^2 L_2 + \alpha'^3 L_3\right)$$

$$L_1 = (\text{Riem})^2 + \ldots \qquad \text{(bos/het)}$$

$$L_2 = (\text{Riem})^3 + \ldots \quad \text{(bos)} \qquad L_2 = (LCS)^2 \quad \text{(het)}$$

$$L_3 = \zeta(3)\,(\text{Riem})^4 + \ldots \qquad \text{(bos/het/type II)}$$

The correction at order $\alpha'^3$

$$S^{(3)} = \frac{\alpha'^3\zeta(3)}{3\cdot 2^{13}}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[(t_8 t_8 + \tfrac{1}{4}\varepsilon_8\varepsilon_8)R^4 + H\text{-terms}\right]$$

# History of $\alpha'$ corrections in supergravity

- R. Metsaev and A. Tseytlin from S-matrix and $\beta$-functions calculations (1987)

$$L^{(1)}_{MT} = -\frac{a+b}{8}\left[R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - \frac{1}{2}H^{\mu\nu\rho}H_{\mu\sigma\lambda}R_{\nu\rho}{}^{\sigma\lambda} + \frac{1}{24}H^4 - \frac{1}{8}H^2_{\mu\nu}H^{2\mu\nu}\right] + \frac{a-b}{4}H^{\mu\nu\rho}C_{\mu\nu\rho}$$

$$(a,b) = \begin{cases} (-\alpha', -\alpha') & \text{bosonic string,} \\ (-\alpha', 0) & \text{heterotic string,} \\ (0,0) & \text{type II string} \end{cases}$$

- Green-Schwarz transformation

$$\delta_\lambda B_{\mu\nu} = \frac{1}{2}\,(a-b)\,\partial_{[\mu}\lambda^{ab}\omega_{\nu]ab}$$

# History of $\alpha'$ corrections in DFT

- Marques, Nuñez (2015) $O(D,D)$ invariant formulation of the first order correction
- Baron, Marques (2020) - a tower of infinitely many higher order corrections but in an implicit form (generalized Bergshoeff-de Roo identification)

$$L = \mathcal{R} + \mathcal{R}^{(0,1)} + b\mathcal{R}^{(1,0)} + a^2\mathcal{R}^{(0,2)} + ab\mathcal{R}^{(1,1)} + b^2\mathcal{R}^{(2,0)} + \dots$$

- Problem: Contains 200(300) terms for the heterotic(bosonic)
- Hronek, Wulff, Zacarías (2022)

$$\begin{aligned}
&-2(\partial^a - F^a)(\partial^b - F^b)\mathcal{M}^{ab} + 2\partial^a F^b \mathcal{M}^{ab} - F_a{}^{bc} F_a{}^{de} \mathcal{M}^{bd}\mathcal{M}^{ce} \\
&+ \tfrac{1}{3}(F^{abc} + aM^{abc})(F^{def} + aM^{def})\mathcal{M}^{ad}\mathcal{M}^{be}\mathcal{M}^{cf} \\
&- \frac{a}{2} R^{ab}{}_{cd} R^{ef}{}_{cd} \mathcal{M}^{ae}\mathcal{M}^{bf} - aF^{abC}\partial_C F^f{}_{de} F^g{}_{de} \mathcal{M}^{af}\mathcal{M}^{bg} \\
&- \frac{a^2}{4} F^a{}_{de} \mathcal{R}^{bc}{}_{de} F^a{}_{fg} \mathcal{R}^{bc}{}_{fg} + \frac{a^2}{2} F^a{}_{fg} \partial^b F^c{}_{fg} F^b{}_{de} \partial^a F^c{}_{de}
\end{aligned}$$

# T-duality

- To make contact with reality we compactify the string $R^{26} \to R^4 \times M$
- In the presence of circular directions the string can wind around them

$$X(\sigma+2\pi) = X(\sigma)+2\pi Rw, \quad w \in \mathbb{Z}, \quad \text{w is the winding number}$$

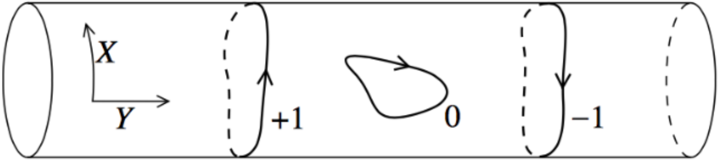


Figure: Taken from Polchinski

- Duality of supergravities (in the presence of background isometries) → Buscher rules

# Double field theory (DFT) formalism

- Idea: Make T-duality a symmetry not a duality.
- Double the coordinates $x^m \to X^M = (\tilde{x}_m, x^m)$.
- $O(D, D)$ is the new structure group and the Lorentz group is doubled to $O(D-1, 1) \times O(1, D-1)$

$$X^M \to O^M{}_N X^N, \quad O \in O(D, D), \quad O \eta O^T = \eta$$

- There are two constant metric, the O(D,D) metric $\eta$ and the Generalized metric $\mathcal{H}$

$$\eta^{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & -\eta^{ab} \end{pmatrix}, \quad \mathcal{H}^{AB} = \begin{pmatrix} \eta_{ab} & 0 \\ 0 & \eta^{ab} \end{pmatrix}$$

$$\eta^{MN} = \begin{pmatrix} 0 & \delta^n_m \\ \delta^m_n & 0 \end{pmatrix}, \quad \mathcal{H}^{MN} = \begin{pmatrix} G_{mn} - B_{mk} G^{kl} B_{ln} & B_{mk} G^{kn} \\ -G^{mk} B_{kn} & G^{mn} \end{pmatrix}$$

# Frame-like flux formalism

$$(G_{mn}, B_{mn}, \phi) \to (E_A{}^M, d)$$

## Definitions and formulas of the doubled fields

- Generalized vielbein:

$$E_A{}^M = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{(+)a}{}_m - e^{(+)an} B_{nm} & e^{(+)am} \\ -e^{(-)}_{am} - e^{(-)n}_a B_{nm} & e^{(-)m}_a \end{pmatrix} .$$

- Generalized dilaton:

$$e^{-2d} = e^{-2\phi}\sqrt{-G}.$$

- Generalized fluxes:

$$F_{ABC} = 3\partial_{[A} E_B{}^M E_{C]M} \,, \qquad F_A = \partial^B E_B{}^M E_{AM} + 2\partial_A d \,.$$

# Unlike ordinary geometry

- Note that $F^{abc}, F_a{}^{bc}, F^a{}_{bc}, F_{abc}, F^a$ and $F_a$ are independent.
- They are manifestly invariant under $O(D,D)$ and generalized diffeomorphism but transform under double Lorentz

$$\delta F^{abc} = 3\partial^{[a}\bar{\lambda}^{bc]} + 3\bar{\lambda}^{[a|d|}F^{bc]d}\,,$$
$$\delta F_a{}^{bc} = \partial_a\bar{\lambda}^{bc} - \underline{\lambda}_{ad}F_d{}^{bc} + 2\bar{\lambda}^{[b|d}F_a{}^{d|c]}$$

- Semi covariant derivatives

$$D_a Y^b = \partial_a Y^b - F_a{}^{bc}Y^c, \qquad D^a Y^b = \partial^a Y^b - \frac{1}{2}F^{abc}Y^c$$
$$D_a Y_b = \partial_a Y_b + \frac{1}{2}F_{abc}Y_c \qquad D^a Y_b = \partial^a Y_b + F^a{}_{bc}Y_c$$

- Analog of the Riemann tensor

$$\mathcal{R}^{ab}{}_{cd} = 2\partial^{[a}F^{b]}{}_{cd} - F^{abe}F^e{}_{cd} + 2F^{[a}{}_{ce}F^{b]}{}_{ed}$$
$$\delta\mathcal{R}^{ab}{}_{cd} = 2\bar{\lambda}^{[a|e|}\mathcal{R}^{|e|b]}{}_{cd} - 2\underline{\lambda}_{[c|e}\mathcal{R}^{ab}{}_{e|d]} - \partial^e\bar{\lambda}^{ab}F^e{}_{cd} - F_e{}^{ab}\partial_e\underline{\lambda}_{cd}$$

# Action, equations and relation to ordinary supergravity

- Action:

$$S = \int \mathrm{d}X e^{-2d} \mathcal{R}$$

$$\mathcal{R} = 4\partial^a F^a - 2F^a F^a + F_a{}^{bc} F_a{}^{bc} + \frac{1}{3} F^{abc} F^{abc}$$

- Equations of motion

$$\mathcal{R} = 0, \quad \partial^a F_b + (\partial_c - F_c) F^a{}_{bc} - F_c{}^{da} F^d{}_{cb} = 0$$

- Returning to supergravity

$$\partial_M = (0, \partial_m), \quad e^{(+)} = e^{(-)} = e$$

# How to get the $\alpha'$ corrections?

## What are we looking for?

- Quantum $\alpha'$ expansion of the DFT action.

$$L = L_0 + \alpha' L_1 + \alpha'^2 L_2$$

- Constructed from $\partial_A, F_A, F_{ABC}$
- Generalized diffeomorphism invariance
- $O(D, D)$ invariance
- Double Lorentz invariance

## How do we get it?

- Via a correction to the double Lorentz transformation
- Solve the following equations

$$\delta' L_0 + \delta L_1 = 0, \quad \delta'' L_0 + \delta' L_1 + \delta L_2 = 0$$

# First order in $\alpha'$

$$L_1 = (\partial^a - F^a)(\partial^b - F^b)M^{ab} - \tfrac{1}{2}\mathcal{R}^{ab}{}_{cd}\mathcal{R}^{ab}{}_{cd} + F^{abC}F^a{}_{de}\partial_C F^b{}_{de}$$
$$- \left(\partial^a F^b - F_c{}^{da}F_c{}^{db} + \frac{1}{2}F^{acd}F^{bcd}\right)M^{ab} + \frac{2}{3}F^{abc}M^{abc}$$

Where

$$M^{ab} = F^a{}_{cd}F^b{}_{dc}, \quad M^{abc} = F^a{}_{de}F^b{}_{ef}F^c{}_{fd}$$

# Second order in $\alpha'$, heterotic case

We split the lagrangian for the heterotic and bosonic case

$$L_2 = a^2\mathcal{R}^{(0,2)} + ab\mathcal{R}^{(1,1)} + b^2\mathcal{R}^{(2,0)}$$

For the heterotic case $b = 0$ we have the full Lagrangian.

$$\begin{aligned} L = & -2(\partial^a - F^a)(\partial^b - F^b)\mathcal{M}^{ab} + 2\partial^a F^b \mathcal{M}^{ab} - F_a{}^{bc} F_a{}^{de} \mathcal{M}^{bd}\mathcal{M}^{ce} \\ & + \frac{1}{3}(F^{abc} + aM^{abc})(F^{def} + aM^{def})\mathcal{M}^{ad}\mathcal{M}^{be}\mathcal{M}^{cf} \\ & - \frac{a}{2} R^{ab}{}_{cd} R^{ef}{}_{cd} \mathcal{M}^{ae}\mathcal{M}^{bf} - aF^{abC}\partial_C F^f{}_{de} F^g{}_{de} \mathcal{M}^{af}\mathcal{M}^{bg} \\ & - \frac{a^2}{4} F^a{}_{de}\mathcal{R}^{bc}{}_{de} F^a{}_{fg} \mathcal{R}^{bc}{}_{fg} + \frac{a^2}{2} F^a{}_{fg}\partial^b F^c{}_{fg} F^b{}_{de}\partial^a F^c{}_{de} \end{aligned}$$

We introduce a new "metric"

$$\mathcal{M}^{ab} = [(\eta + \frac{a}{2}M)^{-1}]^{ab} = \eta^{ab} - \frac{a}{2}M^{ab} + \frac{a^2}{4}(M^2)^{ab} + \dots .$$

We also find the correct transformations.

# Second order in $\alpha'$, bosonic case

$$\mathcal{R}^{(1,1)} = \frac{1}{3}\mathcal{R}^{ab}{}_{cd}\mathcal{R}^{ae}{}_{cf}\mathcal{R}^{be}{}_{df} + \mathcal{O}(F^4)$$

# Higher order corrections

- In previous work (2109.12200) we showed that the known order $\alpha'^3$ supergravity term $\zeta(3)\,(\text{Riem})^4$ cannot be written in DFT.
- Use an improved version of the generalized Bergshoeff-de Roo identification.
- The new metric

$$\mathcal{M}^{ab} = [(\eta + \frac{a}{2}M)^{-1}]^{ab} = \eta^{ab} - \frac{a}{2}M^{ab} + \frac{a^2}{4}(M^2)^{ab} + \dots$$

gives many higher order terms.

# Adding fermions and supersymmetry

- Based on work of Lescano, Nuñez, Rodríguez (2104.09545)
- New fields: gravitino $\psi_a$ and dilatino $\rho$.

$$L = \mathcal{R} - 2\overline{\psi}_a \gamma^b \nabla^b \psi_a - 2\overline{\rho}\gamma^b \nabla^b \rho + 4\overline{\psi}_a \nabla_a \rho$$

Where

$$\nabla^b \psi_a = \partial^b \psi_a - \omega^b{}_{ac}\psi_c - \frac{1}{4}\omega^{bcd}\gamma^{cd}\psi_a, \quad \nabla_A \rho = \partial_A \rho - \frac{1}{4}\omega_A{}^{cd}\gamma^{cd}\rho$$

- Corrected Lorentz symmetry fixed the action only partially we also need supersymmetry.

$$L = \mathcal{R}^{*(0,1)} - 4\overline{\psi}_{cd}\gamma^b \nabla^b \psi_{cd} - 4\overline{\psi}_a \gamma^b \psi_{cd} \overline{D}_a F^b{}_{cd} + \frac{1}{2}\left(\overline{\psi}_a \gamma^b D^c \psi_a\right.$$

$$\left. + \overline{\rho}\gamma^b \partial^c \rho\right) M^{bc} - \frac{1}{4}\left(F^b{}_{ef}\overline{D}^c F^d{}_{fe} + \frac{2}{3}M^{bcd}\right)\left(\overline{\rho}\gamma^{bcd}\rho + \overline{\psi}_a \gamma^{bcd}\psi_a\right)$$

$$+ 4\overline{\psi}_{cd} F^a{}_{cd}\partial^a \rho - \mathcal{R}^{ab}{}_{cd}\overline{\psi}_{cd}\gamma^{ab}\rho + \frac{1}{2}F^b{}_{de}\overline{D}_a F^c{}_{de}\overline{\psi}_a \gamma^{bc}\rho$$

# Conclusions and outlook

- We are currently working on the second order Lagrangian for the supersymmetric case.
- DFT works very well as a simplification tool for many calculations including the first $\alpha'$ corrections
- However it is not able to account for all of the string $\alpha'$ corrections (in the non-compactified case).
- There are some possibilites to go beyond order $\alpha'^2$
- For further interest, recommended talks on youtube by Linus Wulff: "O(d,d) and $\alpha'$-corrections (Linus Wulff)", "Linus Wulff, O(D,D) and string alpha'-corrections"

# Thank you for the attention!

# Appendix: $t_8 t_8$ structure

- The form of the $\mathcal{R}^4$ invariant:

$$
\begin{aligned}
& R^{ab}{}_{ef} R^{bc}{}_{fg} R^{cd}{}_{he} R^{da}{}_{gh} + \tfrac{1}{2} R^{ab}{}_{ef} R^{bc}{}_{fg} R^{cd}{}_{gh} R^{da}{}_{he} \\
& - \tfrac{1}{4} R^{ab}{}_{ef} R^{ab}{}_{fg} R^{cd}{}_{gh} R^{cd}{}_{he} - \tfrac{1}{4} R^{ab}{}_{ef} R^{bc}{}_{ef} R^{cd}{}_{gh} R^{da}{}_{gh} \\
& - \tfrac{1}{8} R^{ab}{}_{ef} R^{cd}{}_{ef} R^{bc}{}_{gh} R^{da}{}_{gh} - \tfrac{1}{8} R_{abef} R^{cd}{}_{fg} R^{ab}{}_{gh} R^{cd}{}_{he} \\
& + \tfrac{1}{16} R^{ab}{}_{ef} R^{cd}{}_{ef} R^{ab}{}_{gh} R^{cd}{}_{gh} + \tfrac{1}{32} R^{ab}{}_{ef} R^{ab}{}_{ef} R^{cd}{}_{gh} R^{cd}{}_{gh} \, .
\end{aligned}
$$

- The other term $\frac{1}{4}\varepsilon_8 \varepsilon_8 R^4$ is a total derivative at leading order in fields.