On Shelah categoricity conjecture, abstract elementary classes and rich families of models

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Abstract

• This works deals with Shelah Categoricity Conjecture (SCC), we are interested on the combinatorics assumed in order that SCC should be valid. There are several examples (studying Shelah amalgamation theorem and its applications in an absense of maximal models), see,e.g. [**1, 2**], as we can see there are several counter examples of the general notion of (SCC), provided and studied by Shelah and Bratt in [**3**]. We see that some versions of the diamond, i.e, for instance, Delvin and Shelah Diamond‘s version are assumed in order to show the validity of the amalgamation property.

Abstract

We know that exactly this kind of the diamond is used for the positive solution of the Whitehead problem (S.Shelah and as well J. Trlifaj), see [**4, 5**], however independently from the main focus of the current paper, following Eklof and Shelah again do find in Trlifaj's work on general projective test module problem - the negative answer, i.e the counter example model in [**6**].

Definition (AEC)

Let us fix the language L, then we will call the class of models/structures C, endowed with a partial order $\leq$ with LS(C) and closed under iso's - AEC if the following holds :

1. $\leq$ is really a substructure embedding.
2. $\leq -$ preserves continuous chains of structures, i.e closed under limits, moreover, this limit is a $\leq -$ upper bound of any of the elements of chain, for any ordinal below the "limit one" and moreover any object of the class is closed under $\leq$-limit.
3. For any $\leq$ - directed triple of structures in C, "being a substructure implies being $\leq$-substructure".

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