Applications of Finsler Geometry in Medical Imaging

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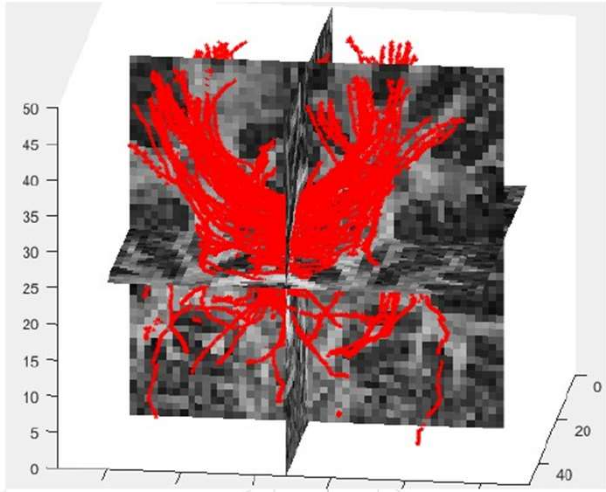
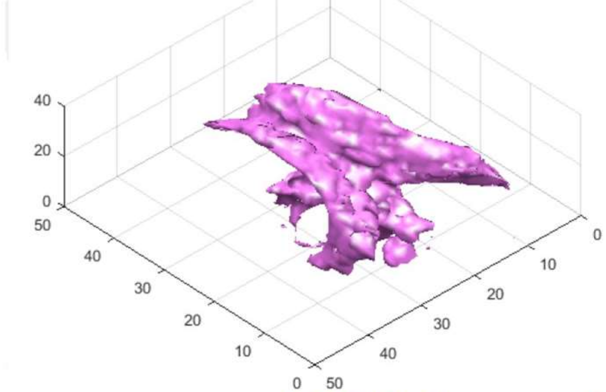
(comments on recent preprints)

Diffusion Tensor Imaging (DTI) relies on the MRI diffusion data, assuming that the diffusion is a Gaussian process. In Brain Imaging, this corresponds to the expectation that each voxel (small element of volume in 3D) contains at most one fiber (the white matter of the brain consists of fibers, while the grey matter is isotropic). The measured data are approximated by a quadratic form at each voxel which, viewed as Riemannian metric, is used to segment the white matter regions and to track the fibers (streamline or second order methods, stochastic or deterministic approach).

With the current technology, many voxels contain more fibers, and the diffusion process is no more Gaussian. Then higher degree tensor data come into the game in the so-called Higher Angle Resolution Diffusion Imaging (HARDI). Here various versions of Finsler metrics related to second and fourth order tensors come are useful (e.g., the 4^{th} root metric given by the 4^{th} order tensor).

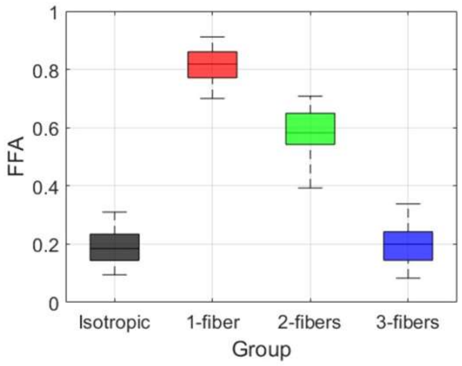


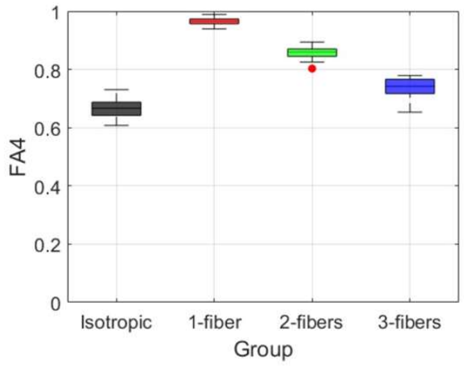


The Finsler Anisotropy scalar

Since the Finsler metrics can be viewed as quadratic forms for each separate direction in the tangent space, we can consider the mean quadratic form for each voxel and consider its fractional anisotropy FFA. Another option is to compute the fractional anisotropy FA4 of the relevant 6x6 matrix form of the 4th degree tensor (the so-called Voigt-Mandel form). Experiments show, that these scalars may distinguish 0-1-2-3 fibers in one voxel (the pictures show the statistics for data with about 10% of noise and many rotations) and this can be well exploited for geodetical fiber tracking by using the approximation of reciprocal values of the measured data (we want to charge the track directions of fastest diffusion the least cost).

$$\mathrm{FFA}(\mathbf{x}) = \mathrm{FA}\left(\frac{1}{p}\sum_{s=1}^{p} g_{ij}(\mathbf{x}, \mathbf{v}_s)\right) \qquad \mathrm{FA4} = \sqrt{\frac{6}{5}\frac{\sum_i(\lambda_i - \lambda_a)^2}{\sum_i \lambda_i^2}},$$





Fiber tracking as input for fast 3D segmentation

A modified hybrid tractography with seed points in voxels with 1 fiber exhausts all of these. The result is then used as input for the 3D active contour segmentation algorithm.

The preprints of these two papers can be found at http://www.math.muni.cz/~slovak/english/publikace.php :

A. Bansal, S. Kaushik, T. Bihonegn, J. Slovak, Automatic Tractography and Segmentation using Finsler Geometry based on Higher-order Tensor Fields , 2022, 18pp.

A. Bansal, S. Kaushik,T. Bihonegn, Denis Baručić, Peter T. While, J. Slovák, Characterizing white matter multi-fiber structure using the Riemann-Finsler framework in HARDI, 2022, 12pp.

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