THE 44th WINTER SCHOOL
GEOMETRY AND PHYSICS

SRNÍ, CZECH REPUBLIC

JANUARY 13–20, 2024

Sponsored by

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ANNOUNCED LECTURES

A. Invited lectures

Ramiro Lafuente: *Einstein manifolds with symmetries*

Shahn Majid: *Quantum Riemannian geometry and applications*

Agustin Moreno: *A modern view on the restricted three-body problem*

Alvaro del Pino Gomez: *Introduction to the h-principle*

Alexander Voronov: *Rational Homotopy Theory, del Pezzo surfaces, and Mysterious Triality*

B. Other lectures

Spyridon Afentoulidis Almpanis: *Dirac cohomology and* Θ*-correspondence for complex dual pairs joint work with G. Liu and S. Mehdi*

Teresa Arias-Marco: *Studying symmetric-like properties on Riemannian manifolds via the spectrum of the Laplace-Beltrami operator*

Samuel Blitz: *A Holographic Approach to Submanifold Geometry*

Eugenia Boffo: *Algebraic structures and particles*

Benedek Bukor: *A less commutative version of quarkonium masses*

Andreas Cap: *Bounded Poincare operators for twisted complexes and BGG complexes*

Darek Cidlinský: *Modes on cube*

Frederik Ďalak: *Geodesic preserving properties of linear connection symmetries*

Thomas De Fraja: *Courant Algebroid Relations and T-duality*

Martin Doležal: *Configuration space of a 3-link snake robot model*

Zdenek Dusek: *Geodesic orbit Finsler* (α, β) *metrics on spheres*

Rita Fioresi: *Generalized Root Systems*

Keegan Flood: *Spencer operators and higher connections in the noncommutative setting*

Roman Golovko: *Instability of Legendrian knottedness and non-regular Lagrangian concordances of knots*

Jan Gregorovič: *On models of 2-nondegenerate CR hypersurfaces*

Leszek Hadasz: *Decomposition of tensor product of affine* sl_2 *highest weight representations and equivalence between CFT models.*

Babak Hassanzadeh Seydei: *Curves on almost Grassmannian structures*

Nikola Herceg: *Metric perturbations in noncommutative gravity*

Goce Chadzitaskos: *Special case of coherent states of the asymmetric harmonic oscillator*

Josef Janyška: *General relativistic Maxwell's equations: jet approach*

Branislav Jurco: *TBA*

Igor Khavkine: *Update on IDEAL characterization of highly symmetric vacuum pp-waves*

Boris Kruglikov: *Relative Differential Invariants*

Andrey Krutov: *Structure of Clifford algebras of isotropy representations associated to symmetric pairs*

Svatopluk Krýsl: *Elliptic complexes of symplectic twistor operators*

Ondřej Kubů: *Symplectic-Haantjes geometry of Hamiltonian integrable systems in magnetic fields*

Ondrej Hulik: *TBA*

Radosław Kycia: *Geometric decomposition and its applications*

Omid Makhmali: *Rolling Finsler surfaces and G2-contact structures*

Michal Marvan: *An integrable class of Chebyshev nets*

Peter Michor: *Higher order symplectic structures on shape spaces of space curves.*

Juan Carlos Morales Parra: *Gauge fixing and Combinatorial Quantization of (Super) Chern-Simons theory*

Filip Moučka: *Symmetric Poisson geometry*

Jiří Nárožný: *Generalised Atiyah's theory of principal connections*

Katharina Neusser: *Compactifications of indefinite 3-Sasaki structures and their quaternionic Kähler quotients*

Réamonn Ó Buachalla: *The noncommutative complex geometry of the full flag manifold of quantum SU(3)*

Pavle Pandzic: *Clifford algebras, symmetric spaces and cohomology rings of Grassmannians*

Marián Poppr: *Cords and submanifolds in* R^3

Filip Pozar: *Noncommutative corrections to black hole entropy*

Tomáš Procházka: *Rational deformations of harmonic oscillator and Bethe equations of 2d free theory*

Martin Schnabl: *On relevant deformations in String Field Theory*

Eivind Schneider: *Invariant divisors and equivariant line bundles*

Jan Slovak: *An example of curved translation principle*

Rudolf Smolka: *Geometric Approach to Graded Vector Bundles*

Vladimír Souček: *A duality for spin 3/2 fields in dimension 6.*

Josef Šilhan: *The gap phenomenon for conformally related Einstein metrics*

Zoran Škoda: *Globalized torsors for Hopf algebras via extensions of corings*

Dennis The: *On 4D split conformal structures with G2-symmetric twistor distribution*

Dominik Trnka: *Operadic Grothendieck Construction and Fibrations*

Rafailia Persefoni Tsiavou: *Dirac operators: Two different approaches to the Seiberg-Witten equations*

Fridrich Valach: *Generalised geometry for the group E7*

Tomáš Vítek: *Brouwer fixed point theorem as a consequence of the Hairy ball problem*

Petr Vlachopulos: *Categorification of the Holographic Principle*

Rikard von Unge: *Root-*$T\bar{T}$ *deformations.*

Jan Vysoký: *Serre-Swan Theorem for Graded Vector Bundles*

Thomas Weber: *Infinitesimal braidings and pre-Cartier bialgebras*

Henrik Winther: *Large automorphism groups of parabolic geometries.*

Lenka Zalabová: *Local control on quaternionic Heisenberg group*

Martin Zika: *Category of Quantum* L_∞ *Algebras*

Alexander Zuevsky: *Canonical extensions of vertex algebra bundles*

ABSTRACTS

Spyridon Afentoulidis Almpanis: *Dirac cohomology and Θ-correspondence for complex dual pairs*
For the last decades, representation theory of Lie groups and algebras has been a very active research topic with a multitude of ramifications and applications. Since the work, in the 1970's, of Parthasarathy and Atiyah-Schmid, Dirac operators have become efficient tools to describe and classify the unitary dual of a real Lie a group G. On the one hand, any irreducible unitary representation occurring in the regular representation $L^2(G)$ can be realized as the Hilbert space of L^2-sections, of some twist of the spin bundle over the Riemannian symmetric space G/K, which belong to the kernel of the associated Dirac operator. Here K is a maximal compact subgroup of G. On the other hand, Dirac cohomology, introduced by Vogan in the late 1990's, defines an invariant which can be used to detect the infinitesimal character of representations (theorem of Huang and Pandžić). Therefore it is important to study the behavior of the Dirac cohomology under functors involved in representation theory.
A useful functor in representation theory of reductive groups is the so-called Θ-correspondence (or the Howe duality). Howe duality relates representations and characters of two Lie groups G_1 and G_2, viewed as closed subgroups of the metaplectic group M such that $Z_M(G_1) = G_2$ and $Z_M(G_2) = G_1$.
In this talk, we will study the behavior of the Dirac cohomology under the Θ-correspondence in the case of complex pairs (G_1, G_2) viewed as real Lie groups. This is joint work with G. Liu and S. Mehdi

Teresa Arias-Marco: *Studying symmetric-like properties on Riemannian manifolds via the spectrum of the Laplace-Beltrami operator*
Symmetric-like properties are those which generalize the local symmetry on Riemannian manifolds. The study of the determination of a geometric property using the information provided by the eigenvalues of the Laplace-Beltrami operator is a problem concerning to the Spectral geometry namely the Audibility problem. In that context, two Riemannian manifolds are said to be isospectral if there exists an operator which intertwines their Laplace-Beltrami operator. If the Riemannian manifolds are compact, the existence of the intertwining operator is equivalent to the fact that both Riemannian manifolds have the same spectrum for the Laplace-Beltrami operator. Moreover, a geometric property is said to be inaudible if there exists a pair of isospectral Riemannian manifolds where that property differs.
In this talk we treat the isospectrality between different pairs of Riemannian manifolds in order to establish the inaudibility of different symmetric-like properties.
Joint work with Jose Manuel Fernandez-Barroso.

Samuel Blitz: *A Holographic Approach to Submanifold Geometry*
The local geometry of higher codimension submanifolds, in the context of both Riemannian and conformal backgrounds, is studied using holographic techniques generalized from the study of hypersurfaces. In both cases, low-order obstructions are found. Nonetheless, by examining special cases of these geometries, we are able to obtain interesting extrinsic invariants.

Eugenia Boffo: *Algebraic structures and particles*
In (closed) string field theory, a dominant role is played by its L_∞ structure and BV formulation, as Barton Zwiebach made clear in 1992. What can we say when one dimension is dropped and thus we are describing "particle field theory"? How much do these algebraic structures unveil

about particles with a supersymmetric world line? This talk will remind of the BV formulation for the string field and its particle counterpart and address their relations/differences.

Benedek Bukor: *A less commutative version of quarkonium masses*
Quarkonium bound states are especially promising candidates to test the probable quantum structure of space- time, since they represent a system with reasonably small characteristic distance. The quantum mechanical interaction between the quarks is heuristically described by the Cornell potential. Here, we insert this system in a 3-dimensional rotationally invariant space which is composed of concentric fuzzy spheres of increasing radius called the fuzzy onion in order to extract some consequences of the non-trivial structure on its properties. The talk will be based on joint work with Juraj Tekel, arXiv:2209.09028.

Andreas Cap: *Bounded Poincare operators for twisted complexes and BGG complexes*
This talk reports on recent joint work with K. Hu (Edinburgh). I'll start by reviewing the concept of Poincare operators and available results for Sobolev-de Rham complexes of bounded Lipschitz domains in R^n. We extend these results to projective and conformal BGG complexes and the corresponding twisted de Rham complexes for the flat Riemannian metric on such domains. I will also discuss Some applications, in particular towards finite element methods.

Darek Cidlinský: *Modes on cube*
In this talk, we will show how to get the eigenvalues and eigenvectors of the Laplacian (i. e. how to solve the Helmholtz equation) on the surface of a cube. Using group theory, we will extract enough information about the modes to be able to get explicit solutions for them using a simple method that can be extended to other objects as well (some regular polyhedra like the octahedron and icosahedron and maybe more).

Frederik Ďalak: *Geodesic preserving properties of linear connection symmetries*
We present a brief summary of continuous transformations of a smooth manifold that preserve the geodesic curves. We introduce the Lie derivative of linear connection as a tool to study linear connection symmetries and projective transformations. We then examine the behavior of geodesic curves on a manifold under these transformations.

Thomas De Fraja: *Courant Algebroid Relations and T-duality*
We develop a new approach to T-duality based on Courant algebroid relations which subsumes the usual T-duality as well as its various generalisations. We introduce the relation associtated to Courant algebroid reduction, and describe how to transfer the information of generalised metrics through the reduction, via transverse generalised metrics and isometries. This is used to construct T-dual backgrounds as generalised metrics on reduced Courant algebroids which are related by a generalised isometry. There is a existence and uniqueness result for generalised isometric exact Courant algebroids coming from reductions.

Martin Doležal: *Configuration space of a 3-link snake robot model*
I study a configuration space of a 3-link snake robot model moving in a plane, which is one of typical models studied in geometric control theory. The configuration space is 5 dimensional and allowed movements of the snake make a rank 2 bracket-generating distribution on it with a growth vector (2,3,5) (in a regular point). One can find vector fields generating this distribution in such a way that they also generate finite dimensional Lie algebra over reals. Thanks to that, it can be completed to a model locally with a Lie group structure and it helps us to determine symmetries also for the original model.

Zdenek Dusek: *Geodesic orbit Finsler* (α, β) *metrics on spheres*
Recently it was proved that all homogeneous Finsler (α, β) metrics F based on geodesic orbit Riemannian metric α are also geodesic orbit metrics. This fact will be illustrated by explicit construction of geodesic graphs on spheres, which leads to an alternative classification of geodesic orbit Finsler (α, β) metrics F on spheres. Interesting examples appear, for which isometry group of geodesic orbit metric α has to be extended to reach the geodesic orbit property of F. This is a joint work with Teresa Arias Marco.

Rita Fioresi: *Generalized Root Systems*
In this talk (joint work with I. Dimitrov, Queens U.) we introduce the category of generalized root systems. The notion of ordinary root systems is the key to understand Lie theory and its many generalizations (contragredient superalgebras, affine, Kac-Moody (super) algebras etc). However, such notion is "rigid", it does not behave reasonably under quotients and moreover lacks of a unified treatment, that is definitions and results are usually confined to the realm of application. The rigidity of ordinary root systems stems from their invariance under the action of the Weyl group. Once we abandon the notion of Weyl group as we know it, we can look for another definition of root systems that is able to take into account all examples mentioned above and more. For example, the systems stemming from the eigenspace decomposition with respect to a non maximal toral subalgebra (Kostant root systems). They play a key role in the classification of the complex structures on the symmetric space G/K, for K and non maximal torus. This is a generalization of the hermitian symmetric spaces theory. In this talk we give an effective way to compute bases for generalized root systems, which are quotients of Lie algebra ones and we classify all root systems of rank two up to combinatorial equivalence finding 16 such.

Keegan Flood: *Spencer operators and higher connections in the noncommutative setting*
Our generalization of the notion of jet functors from the setting of classical differential geometry to the setting of a unital associative algebra equipped with a differential calculus allows one to extend many jet related constructions to this noncommutative setting, given the imposition of mild homological conditions on the differential calculus. In this talk we will discuss a few results along these lines involving principal symbols, Spencer operators, higher connections, and their interplay.

Roman Golovko: *Instability of Legendrian knottedness and non-regular Lagrangian concordances of knots*
We will show that family of smoothly non-isotopic Legendrian pretzel knots from the work of Cornwell–Ng–Sivek that all have the same Legendrian invariants as the standard unknot have front-spuns that are Legendrian isotopic to the front-spun of the unknot. Besides that, we construct examples of Lagrangian concordances between Legendrian knots that are not regular, and hence not decomposable – these are probably the first known such examples – and we discuss their behaviour under front-spinning. This is joint work with Georgios Dimitroglou Rizell.

Jan Gregorovič: *On models of 2-nondegenerate CR hypersurfaces*
I will talk about joint work with D. Sykes about models of uniformly 2-nondegenrate CR hypersurfaces. We construct all (up to equivalence) weighted homogeneous (with respect to a natural weighting system) rigid uniformly 2-nondegenerate CR hypersurfaces whose holomorphic infinitesimal symmetries generate a transitive action on their Levi leaf spaces. For hypersurfaces in C^4, we obtain a full classification of such CR hypersurfaces and identify the homogeneous models among them that realize particular CR invariants termed modified symbols.

Leszek Hadasz: *Decomposition of tensor product of affine sl_2 highest weight representations and equivalence between CFT models.*

During my presentation I will present an explicit decomposition of $\widehat{\mathfrak{sl}}_{2,k} \otimes \widehat{\mathfrak{sl}}_{2,1}$ highest weight representations with irrational level k onto direct sum of highest weight represtations of the $\widehat{\mathfrak{sl}}_{2,k+1} \otimes \mathrm{Vir}$ algebra, along with some new results for the $\widehat{\mathfrak{sl}}_{2,k}$ singular vectors. The obtained results prove certain equivalences between two dimensional CFT models, can be extended to higher rank algebras and applied to generalize the AGT relation.

Babak Hassanzadeh Seydei: *Curves on almost Grassmannian structures*

We study curves in manifolds with almost Grassmannian structure or AG-structure. In this case tangent bundle is identified by tensor product of two auxiliary bundles of dimension p and q. Here we study special case p=2 and q=4 and using suitable tractor bundle we find preferred parametrizations.

Nikola Herceg: *Metric perturbations in noncommutative gravity*

We apply the formalism of noncommutative differential geometry, based on Hopf algebra and Drinfeld twist, in finding perturbations of a background metric in noncommutative gravity. In this bottom-up approach, a natural candidate for an Einstein manifold emerges, with R-matrix and braided symmetry playing a central role. Axial and polar perturbations of the Schwarzschild background are discussed.

Goce Chadzitaskos: *Special case of coherent states of the asymmetric harmonic oscillator*

We have constructed formal coherent states for an asymmetric harmonic oscillator, where the asymmetry parameter is the square root of ratio of spring constants. These states generally do not satisfy all the required properties for coherent states. During the time development, coherent states introduced in this way become decoherent generally. For some specific parameters it is possible to construct coherent states on a subspace of the Hilbert space of eigenstates. These coherent states preserve coherence during time evolution.

Josef Janyška: *General relativistic Maxwell's equations: jet approach*

The phase space of general relativistic spacetime can be defined as the 1st jet space of motions. A Lorentzian metric and an electromagnetic 2-form allow to define naturally a geometric structure of the phase space. This structure makes it possible to describe the classical mechanics of spacetime. As an example, we will give Maxwell's equations and their observed expressions.

Branislav Jurčo: *TBA*

Igor Khavkine: *Update on IDEAL characterization of highly symmetric vacuum pp-waves*

Vacuum pp-waves are a class of 4-dimensional solutions of the Einstein equations with several interesting properties. In particular, they belong to the class of spacetimes with vanishing scalar invariants. Thus, they are expected to be a challenging case for finding IDEAL characterizations, which aim to identify a particular spacetime isometry class by a list of tensor equations covariantly constructed from the metric and the curvature. I will discuss some of the obstacles and how they were overcome with a slight modification of the IDEAL approach. The result is a complete classification and characterization of the isometry classes of vacuum pp-waves with at least two independent Killing vectors. Based on joint work with D. McNutt and L. Wylleman.

Boris Kruglikov: *Relative Differential Invariants*

Relative invariants describe singular fibers of orbit foliations. In geometric settings they are known as fundamental invariants. Upon prolongations they give rise to relative differential invariants. We will discuss the theory of such invariant in a global setting. For a pseudogroup action in jets or on differential equations, we will present a finite-generation property for the algebra of polynomial relative differential invariants and demonstrate it on some examples. Joint work with Eivind Schneider.

Andrey Krutov: *Structure of Clifford algebras of isotropy representations associated to symmetric pairs*
Let $(\mathfrak{g}, \mathfrak{k})$ be a symmetric pair and $\mathfrak{p}$ be the corresponding isotropy representation of $\mathfrak{k}$, namely, $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$. Let $\mathfrak{h}$ be a Cartan subalgebra of $\mathfrak{g}$ such that $\mathfrak{t} := \mathfrak{k} \cap \mathfrak{h}$ is a Cartan subalgebra of $\mathfrak{k}$. Set $\mathfrak{a} := \mathfrak{h} \cap \mathfrak{p}$. Assume that $\mathfrak{g}$ admits a non-degenerate symmetric bilinear form B. Then the restriction of B to $\mathfrak{k}$ and $\mathfrak{p}$ is also non-degenerate which allows us to consider the Clifford algebra $Cl(\mathfrak{p})$ of the isotropy representation. One can show that the subalgebra of $\mathfrak{k}$-invariants in $Cl(\mathfrak{p})$ is isomophic to the tensor product of the Clifford algebra $Cl(\mathfrak{a})$ and the characteristic subalgebra A. In the talk we will discuss the structure of the algebra A. This is joint work with K. Calvert, K. Grizelj, and P. Pandžić.

Svatopluk Krýsl: *Elliptic complexes of symplectic twistor operators*
Symplectic twistor operators are analogues of Lorentzian twistor operators of Penrose. In opposite to Riemannian and Lorentzian spinors, symplectic spinors are vector spaces of infinite dimension. We introduce a resolution of symplectic spinors for symplectic manifolds with a symplectic torsion-free connection, whose complexified tangent bundles' Chern classes are even and whose symplectic Weyl-tensor vanishes. We give a tool (based on a Schur-Weyl-type duality), that enables to prove that specific symplectic spinor-valued wedge forms on the considered manifolds are elliptic.

Ondřej Kubů: *Symplectic-Haantjes geometry of Hamiltonian integrable systems in magnetic fields*
Despite significant effort in the new millennium, integrable systems immersed in magnetic fields are not well understood, mainly because the 1:1 correspondence with separation of variables of Hamilton-Jacobi (HJ) equation is broken. Moreover, the standard theory of separation of variables on the configuration space requires to fix the gauge in a somewhat ad hoc manner.
In this talk we study the geometry of physically relevant integrable systems with magnetic fields using the recently proposed formalism based on symplectic-Haantjes manifolds. In addition to the theoretical insight, the geometry naturally fixes the gauge thanks to its definition on the full phase space. We also obtain a new family of integrable systems on curved manifolds by generalizing the obtained geometries.

Ondrej Hulik: *TBA*

Radosław Kycia: *Geometric decomposition and its applications*
I will show how to use the geometric decomposition, i.e., split of differential forms module over the star-shaped set into exact/closed and antiexact parts, to solve some problem from differential geometry, variational calculus, and algebraic topology. The talk is motivated by [1], [2], [3], and some recent results.
Bibliography:
[1] R.A. Kycia, J. Šilhan, "Inverting covariant exterior derivative", arXiv:2210.03663. [2] E.A. Kycia, "The Poincare Lemma for Codifferential, Anticoexact Forms, and Applications to Physics". Results Math 77, 182 (2022). [3] R.A. Kycia, "The Poincare Lemma, Antiexact Forms, and Fermionic Quantum Harmonic Oscillator". Results Math 75, 122 (2020).

Ramiro Lafuente: *Einstein manifolds with symmetries*
In this lecture series, I plan to give a gentle introduction to Riemannian geometry with symmetries, with a focus on the Ricci curvature, and the Einstein and Ricci soliton equations. We will begin with the case of homogeneous manifolds, and then move on to studying the Einstein equation on principal bundles. We will finally focus on the recent resolution of the 1975 Alekseevskii conjecture, and discuss future directions and open problems.

Shahn Majid: *Quantum Riemannian geometry and applications*
We will describe the state of the art in an approach to noncommutative geometry based on quantum metrics g in the tensor square of Ω^1, the space of 1-forms over a possibly noncommutative algebra A, and bimodule connections. In Lecture 1 we will then focus on applications to physics when A is finite dimensional, namely to baby models of quantum gravity and to elementary particle physics via Kaluza-Klein ideas. In Lecture 2 we will focus on quantum geodesics flows based on a bicategory of A-B differentiable bimodules with applications to quantum mechanics and to graphs. In Lecture 3 we will focus on a constructive (but not the most general) approach to quantum jet bundles. The lectures will be based on recent joint work and will build on the general formalism in the 2020 book with E.J. Beggs.

Omid Makhmali: *Rolling Finsler surfaces and G2-contact structures*
The notion of rolling for two Riemannian surfaces is well-known which is encoded in terms of a (2,3,5)-distribution on their 5-dimensional configuration space. Using a reformulation of the rolling problem, we will see how the notion of rolling can be extended to Finsler surfaces and in certain cases results in a G2-contact structure.

Michal Marvan: *An integrable class of Chebyshev nets*
We study curve nets satisfying a R-linear relation between the Schief curvature of the net and the Gauss curvature of the supporting surface. We write an $\mathfrak{sl}(2)$-valued zero-curvature representation, associated with an elliptic spectral curve and reveal two cases when the curve degenerates. In the particular case when the curvatures are proportional (concordant nets) we find a correspondence to pairs of pseudospherical surfaces of equal negative constant Gaussian curvatures. The construction generalises the well-known correspondence between translation surfaces and pairs of curves. In addition, we give an overview of curve net invariants and relations among them.

Peter Michor: *Higher order symplectic structures on shape spaces of space curves.*
For $c \in \mathrm{Imm}(S^1, R^3)$ the 2- form $\Omega_c^{MW}(h,k) = 3\int_{S^1} \det(D_s c, h, k) ds$ induces the Marsden-Weinstein symplectic structure on the shape space $\mathrm{Imm}(S^1, R^3)/\mathrm{Diff}(S^1)$, corresponding to a Kähler structure. The Hamiltonian flow for the length functional is the binormal flow. In this talk I will present other symplectic structures related to this.

Juan Carlos Morales Parra: *Gauge fixing and Combinatorial Quantization of (Super) Chern-Simons theory*
The moduli space of (super)flat connections over a Riemann surface $\Sigma_{g,n}$ could be realized as a constrained Poisson structure, with brackets defined in terms of a (super)classical r-matrix of the underlying (super)Lie algebra. After gauge fixing the constraints, the derived Dirac brackets resemble the original ones, but are now defined in terms of a (super)classical dynamical r-matrix. Following Fock-Rosly approach, we explain how these gauge-fixed structures could be understood as "special"decorated character varieties and so, following Alekseev-Grosse-Schomerus combinatorial quantization scheme, we will describe how (super)quantum dynamical R-matrices

and (super)Quantum Dynamical Groups appear naturally at the quantum level, using decorated SL(2, C) and GL(1|1) character varieties over punctured spheres as an illustrative examples. (This talk is based on joint work with Bernd Schroers).

Agustin Moreno: *A modern view on the restricted three-body problem*
In this series of lectures, I will revisit the classical restricted three-body problem, going back to Newton and Poincaré, from the modern perspective of contact and symplectic geometry. I will touch on open book decompositions, fixed-point theory of Hamiltonian maps, and pseudo-holomorphic curves, following recent advances in the context of Hamiltonian dynamics. Part of the story will be based on joint work with Otto van Koert.

Filip Moučka: *Symmetric Poisson geometry*
A Poisson manifold is a generalization of the notion of phase space from Hamiltonian mechanics. It is a manifold endowed with a skew-symmetric bivector field such that the Schouten bracket of the bivector field with itself vanishes. We discuss what happens, when you consider a symmetric bivector field instead of a skew-symmetric one.

Jiří Nárožný: *Generalised Atiyah's theory of principal connections*

Katharina Neusser: *Compactifications of indefinite 3-Sasaki structures and their quaternionic Kähler quotients*
We will show that 3-Sasaki manifolds can be equivalently viewed as projective structures equipped with a certain type of quaternionic unitary holonomy reduction of their canonical Cartan connection. Moreover, we will see that a general quaternionic unitary holonomy reduction of a projective structure decomposes the base manifold into union of submanifolds with certain induced geometric structures. As an application, we will describe natural geometric compactifications of (suitably) complete indefinite 3-Sasaki structures and their quaternionic Kähler quotients. This talk is based on joint work with A. Rod Gover and Travis Willse.

Réamonn Ó Buachalla: *The noncommutative complex geometry of the full flag manifold of quantum SU(3)*
We construct a q-deformation of the Dolbeault double complex of the full quantum flag manifold of SU(3). This extends the Heckenberger-Kolb differential calculus of the quantum projective plane, and the recently constructed q-deformed anti-holomorphic complex of the full quantum flag. We present a number of new phenomenon which do not occur in the Hermitian symmetric setting, such as connections with torsion and non-integrable almost-complex structures.

Pavle Pandzic: *Clifford algebras, symmetric spaces and cohomology rings of Grassmannians*
We study various kinds of Grassmannians or Lagrangian Grassmannians over R, C or H, all of which can be expressed as G/P where G is a classical group and P is a parabolic subgroup of G with abelian unipotent radical. The same Grassmannians can also be realized as (classical) compact symmetric spaces G/K. We give explicit generators and relations for the de Rham cohomology rings of $G/P \cong G/K$. At the same time we describe certain filtered deformations of these rings, related to Clifford algebras and spin modules. While the cohomology rings are of our primary interest, the filtered setting of K-invariants in the Clifford algebra actually provides a more conceptual framework for the results we obtain. This is joint work with Kieran Calvert and Kyo Nishiyama.

Alvaro del Pino Gomez: *Introduction to the h-principle*
Given a family R of geometric structures (for instance: symplectic structures, foliations, or metrics

with positive scalar curvature) and a manifold M, we can ask ourselves: "what is the homotopy type of the space of geometric structures of type R on M?"Concretely, we can try to determine how many path-components it has, whether these have non-trivial loops, and so on.
The homotopy principle (h-principle for short) is the field of Differential Topology that tries to address these questions. A priori, it may seem unexpected that one can say anything in general, since different families R may have quite different properties. However, Gromov observed that there are certain patterns, shared by many different families, that allow us to tackle them in a uniform manner. For instance, one of the techniques developed by Gromov, called convex integration, applies to all geometric structures described by a differential relation satisfying the so-called "ampleness"condition.
Throughout its history (starting from the 1950s), the h-principle has had an impact on the study of immersions, embeddings, symplectic structures, contact structures, foliations, metrics satisfying various curvature constraints, and many other geometric structures (and recently, on the construction of low-regularity solutions of PDEs coming from fluid dynamics). In many cases, it has been instrumental in guiding further research.
The goal of the course is to (1) introduce the language necessary to discuss and tackle h-principle results, (2) sketch some of the classic techniques, (3) touch upon more recent results and open questions.

Marián Poppr: *Cords and submanifolds in R^3*
The study of the algebra of cords with endpoints on the knot in R^3 gives us a powerful knot invariant. In fact, originaly due to Lenhard Ng, this algebra detects the unknot. First, we will give an overview of this invariant in the Morse-theoretical setup. Then we enlight the origin of a bit mysterious skein relations via J-holomorphic curves. Lastly, we will discuss the extension of the theory to cords on knotted tori in R^3.

Filip Pozar: *Noncommutative corrections to black hole entropy*
Noncommutative geometry is an established potential candidate for including quantum phenomena in gravitation. We outline the formalism of Hopf algebras and its connection to the algebra of infinitesimal diffeomorphisms. Using a Drinfeld twist we deform spacetime symmetries, algebra of vector fields and differential forms leading to a formulation of noncommutative Einstein equations. We study a concrete example of charged BTZ and RN spacetime and deformations steaming from the so called angular twist. The entropy of the noncommutative black hole is obtained using the brick-wall method. We provide the method to calculate corrections to the Bekenstein-Hawking entropy in higher orders in WKB, but we present the final result in the lowest WKB order. The result is that even in the lowest order in WKB, the entropy, in general, contains higher powers in h, and it has logarithmic corrections. In contrast, such logarithmic corrections in the commutative setup appear only after the quantum effects are included through higher order WKB corrections or through higher loop effects. Our analysis thus provides further evidence towards the hypothesis that the noncommutative framework is capable of encoding quantum effects in curved spacetime.

Tomáš Procházka: *Rational deformations of harmonic oscillator and Bethe equations of 2d free theory*
The Virasoro algebra, the underlying symmetry algebra of two-dimensional conformal field theory, as well as its high spin extensions, admits various integrable structures - infinite collections of commuting quantities which are analogous to Cartan subalgebras of simple Lie algebras. The spectrum of these commuting quantities is rather non-trivial, but one way to describe it is using

Bethe ansatz equations. For particular (local) choice of the commuting quantities, some of the solutions of BAE are singular and the question is how to characterize their singular behavior. The answer involves combinatorics of partitions (Young diagrams), solvable rational deformations of quantum mechanical harmonic oscillator as well as special rational solutions of the classical KP hierarchy of integrable partial differential equations and Adler-Moser polynomials.

Martin Schnabl: *On relevant deformations in String Field Theory*
I will summarize recent advances on understanding relevant deformations in 2D CFTs from the perspective of Open and Closed String Field Theory.

Eivind Schneider: *Invariant divisors and equivariant line bundles*
Analytic hypersurfaces of a complex manifold M is described by divisors. Any divisor gives rise to a line bundle over M, and the corresponding invariant hypersurface is described by the vanishing of a nontrivial section of this bundle. When M is equipped with a Lie algebra of vector fields, one of the main tasks is to find the $\mathfrak{g}$-invariant hypersurfaces in M. For this we define $\mathfrak{g}$-invariant divisors, a global counterpart to the well-studied scalar relative invariants. We discuss how the invariant divisors relate to $\mathfrak{g}$-equivariant line bundles, and show several aspects of the theory of invariant divisors with computational examples.
This work, which is joint with B. Kruglikov, was motivated by applications to jet bundles, in particular by the task of finding invariant differential equations. Some of these applications are discussed in his talk on relative differential invariants.

Jan Slovak: *An example of curved translation principle*
As well known, a choice of particular geometry expresses the expected symmetries of our mathematical models, the invariant linear differential operators often form nice sequences, and linearized theories in Physics are often identifying certain subcomplexes there. For the Klein geometries, all this has got a well understood algebraic formulation in terms of induced modules and their morphisms and, in the case of the parabolic geometries, we enjoy the rich theory of Verma modules and the Jantzen-Zuckermann translation principle there. Curved versions of such considerations were introduced about 30 years ago, but they cover only a few examples of the relevant Cartan geometries so far. The talk will report some recent results related to specific Grassmannian generalizations of conformal Riemannian geometries. This is a joint (still unfinished) work with Vladimir Soucek.

Rudolf Smolka: *Geometric Approach to Graded Vector Bundles*
In the theory of $\mathbb{Z}$-graded manifolds one usually defines the so-called graded vector bundles first by their (graded) sheaf of sections, and then proceeds to construct the underlying graded manifold. In this talk we will show a "geometric"definition much more akin to the usual approach to vector bundles in ordinary (non-graded) differential geometry. Some examples and properties of graded vector bundles will also be discussed.

Vladimír Souček: *A duality for spin 3/2 fields in dimension 6.*
Main topic of the lecture is a Howe type duality for massless fields of spin 3/2 in its Euclidean version. Main problems treated in the lecture are a description of fields which need to be considered in the study of massless fields of spin 3/2; a suitable choice of equations they should satisfy; irreducibility of homogeneous solutions of massless field equations; the Fischer decomposition and the Howe duality for such fields.

Josef Šilhan: *The gap phenomenon for conformally related Einstein metrics*
We determine that submaximal dimensions of almost Einstein scales and normal conformal Killing

fields for conformal manifolds. That is, we determined maximal dimensional of corresponding overdetermined systems of PDEs assuming the connected conformal manifold is not conformally flat. The upper bound we obtain is sharp and we provide specific examples which realize this upper bound in all signatures.

Zoran Škoda: *Globalized torsors for Hopf algebras via extensions of corings*
We present a formalism for torsors with Hopf algebra in place of a structure group where both the total and base space are given by corings which in turn represent flat covers of noncommutative spaces; the coring for the total space in addition has a coaction by a Hopf algebra. This construction generalizes Hopf-Galois extensions of algebras to which it reduces in the case of trivial covers. For cleft Hopf-Galois extensions the theory leads to Čech-like data, which can be compared for different covers.

Dennis The: *On 4D split conformal structures with G2-symmetric twistor distribution*
In their 2013 article, An and Nurowski considered two surfaces rolling on each other without twisting or slipping, and defined a twistor distribution (on the space of all real totally null self-dual 2-planes) for the associated 4D split signature conformal structure. If this conformal structure is not anti-self dual, then the twistor distribution is a (2,3,5)-distribution, and An-Nurowski identified interesting rolling examples where it achieves maximal, i.e. G2, symmetry. Relaxing the rolling assumption, a similar construction can be made for any 4D split conformal structure, and my talk will discuss a broader classification of examples where such exceptional symmetry for the twistor distribution is achieved. (Joint work with Pawel Nurowski and Katja Sagerschnig.)

Dominik Trnka: *Operadic Grothendieck Construction and Fibrations*
I will recall the concept of a fibration, which is a very important object in algebraic topology and category theory. In category theory, every functor gives a fibration via the famous Grothendieck construction and every fibration assembles into a functor, providing an equivalence of these two notions. Then I will introduce operadic categories, present some examples and accomodate fibrations and Grothendieck construction into the operadic setting. Operads and operad-like structures are tools used to handle homotopy theory of various algebras both in mathematics and physics. Emerging from this, operadic categories are a recent framework designed to handle operads and operad-like structures, relations between them, and their homotopy theory, which in turn provides valuable information for homotopy theory of algebras.

Rafailia Persefoni Tsiavou: *Dirac operators: Two different approaches to the Seiberg-Witten equations*
A Dirac operator D on a smooth manifold M is built to play the role of the operator-square root of the Laplacian Δ. Considering solutions of $D\psi = 0$, we examine the "harmonicity"of spinors ψ. Seiberg-Witten equations couple $D\psi = 0$ with an initial condition on the curvature of the (Riemannian) manifold M. In terms of Differential Geometry, if $\mathcal{S} = \mathcal{S}^+ \oplus \mathcal{S}^-$ is the spinor bundle over M, the Dirac operator $D_\nabla : \Gamma(\mathcal{S}^+) \to \Gamma(\mathcal{S}^-)$ is defined as the composition of the covariant derivative ∇ with the Clifford multiplication cl

$$D_\nabla : \Gamma(\mathcal{S}^+) \xrightarrow{\nabla} \Gamma(T^*M \otimes \mathcal{S}^+) \xrightarrow{\mathrm{cl}} \Gamma(\mathcal{S}^-).$$

On the other hand, in terms of Representation Theory, the Dirac operator is defined on the sections of the homogeneous bundle $G \times_H (S \otimes E) \to M$, considering M as a homogeneous space G/H, i.e.

$$\mathcal{D} : \Gamma(G/H, S \otimes E_{s,\rho}) \to \Gamma(G/H, S \otimes E_{s,\rho}).$$

We discuss the differences and similarities between the two definitions and their interpretation.

Fridrich Valach: *Generalised geometry for the group E7*
Various aspects of string theory (especially those pertaining to its massless sector) can be conveniently described using the so-called generalised geometry. Embedding the setup into the framework of Courant algebroids, one obtains in addition a useful handle for the study of dualities. The corresponding story in the case of (reductions of) M-theory, called exceptional generalised geometry due to the presence of exceptional U-duality groups, is much less developed. Only recently it was understood what is the right generalisation of Courant algebroids in this context. However, due to issues related to the dual graviton, this understanding works only for groups with rank<7. I will describe what is the relevant type of structure for the case n=7, and how it relates to the Poisson-Lie U-duality and consistent truncations of M-theory. This is a joint work with O. Hulik, E. Malek, and D. Waldram.

Tomáš Vítek: *Brouwer fixed point theorem as a consequence of the Hairy ball problem*
Brouwer fixed point theorem is one of the key results about topology of Euclidean spaces, alongside the hairy ball problem. In this talk I will speak about their relationship and their non-standard proofs that do not use any notions from algebraic topology, such as homology or a degree of a map, making them basically elementary.

Petr Vlachopulos: *Categorification of the Holographic Principle*
The holohraphic principle as a straightforward consequence of the quantum information theory of separable systems, provides a basis for the theories of measurement, scatering amplitudes and various dualities between string theories and conformal field theories. Many apparently equivalent principles appear in other disciplines such as computer science, neuroscience or life sciences. However, despite its frequent applications, the mathematical foundations of this principle are weak or almost non-existent. By adapting the perspective of categorification, we try to potentially axiomatize the holographic principle and thus justify its place, as an apparent law of physics that stands by itself.

Rikard von Unge: *Root-$T\bar{T}$ deformations.*
We discuss some attempts to define the Root-$T\bar{T}$ operator in Conformal field theory and discuss the consequences in some simple examples.

Alexander Voronov: *Rational Homotopy Theory, del Pezzo surfaces, and Mysterious Triality*
Mysterious duality was discovered by Iqbal, Neitzke, and Vafa in 2002 as a convincing, yet mysterious correspondence between certain symmetry patterns in dimensional reductions of M-theory and del Pezzo surfaces, both governed by the root system series E_k. It turns out that the sequence of del Pezzo surfaces is not the only sequence of objects in mathematics which gives rise to the same E_k symmetry pattern. There is a sequence of topological spaces, starting with the four-sphere S^4, and then its iterated cyclic loop spaces $L_c^k S^4$, within which we see the E_k symmetry pattern via rational homotopy theory. For this sequence of spaces, the correspondence between its E_k symmetry pattern and that of dimensional reductions of M-theory is no longer a mystery, as each space $L_c^k S^4$ is naturally related to a certain dimensional reduction of M-theory via identification of the equations of motion of $(11-k)$-dimensional supergravity as the defining equations of the Sullivan minimal model of $L_c^k S^4$. This gives an explicit duality between rational homotopy theory and physics. Thereby, Iqbal, Neitzke, and Vafa's mysterious duality between algebraic geometry and physics is extended to a triality involving algebraic topology, with the duality between topology and physics made explicit and demystified. The mystery is

now transferred to the mathematical realm as duality between algebraic geometry and algebraic topology. In the lectures, I will give an introduction to rational homotopy theory and del Pezzo surfaces and explain how the patterns of E_k symmetry arise in these contexts as well as in supergravity. Based on arXiv:2111.14810 and arXiv:2212.13968, with Hisham Sati.

Jan Vysoký: *Serre-Swan Theorem for Graded Vector Bundles*
In ordinary geometry, Serre-Swan theorem relates a geometrical definition of vector bundles to finitely generated projective modules. This fundamental result allows one to work with vector bundles in an entirely algebraic way.
Graded vector bundles over graded manifolds can be introduced wither as a particular graded manifolds, or as sheaves of modules (of their sections). It is expected that they correspond to finitely generated projective modules in a similar fashion. However, since graded vector bundles cannot be described by their fibers, one cannot use the standard arguments to prove the theorem. Basic definitions and a sketch of the proof are presented.

Thomas Weber: *Infinitesimal braidings and pre-Cartier bialgebras*
We propose an approach to infinitesimal braidings which applies to arbitrary braided monoidal categories. The motivating idea is to understand an infinitesimal braiding as a first order deformation of a given braiding. We call braided monoidal categories endowed with an infinitesimal braiding 'pre-Cartier', because they generalize previously studied Cartier categories. It is the main goal of this talk to present the algebraic structure on coquasitriangular bialgebras which characterizes infinitesimal braidings on their categories of comodules. It turns out that this pre-Cartier bialgebra structure corresponds to Hochschild 2-cocycles which satisfy a deformed version of the quantum Yang-Baxter equation, while it gives rise to Hochschild 2-coboundaries in the Cartier cotriangular Hopf algebra framework. We discuss explicit examples on q-deformed GL(2) and Sweedler's Hopf algebra. As main results we provide an infinitesimal FRT construction and a Tannaka-Krein reconstruction theorem for pre-Cartier coquasitriangular bialgebras. The former admits canonical non-trivial solutions and thus induces non-trivial infinitesimal R-forms on all FRT bialgebras. The talk is based on a collaboration with Ardizzoni, Bottegoni and Sciandra.

Henrik Winther: *Large automorphism groups of parabolic geometries.*
We discuss maximal and submaximal global symmetry dimensions, i.e. dimensions of automorphism groups of parabolic geometries. This problem in the general context is largely algebraic. Under an additional restriction of compactness of the group the problem is well-studied. If we impose an additional restriction of compactness of the manifold with parabolic geometry, the problem becomes much more complicated. Joint work with Boris Kruglikov

Lenka Zalabová: *Local control on quaternionic Heisenberg group*
We describe the quaternionic Heisenberg group in dimension seven and study the corresponding invariant control system. We describe the geodesics of the system and study the action of symmetries of the corresponding sub-Riemannian structure on them. We find corresponding Maxwell points given by the symmetries.

Martin Zika: *Category of Quantum L_∞ Algebras*
Categories of relations including Weinstein's (linear) symplectic category are known to provide semantics convenient for quantum theory. We apply this philosophy to quantum L-infinity algebras, which give a homotopy algebraic framework for perturbative quantum field theories. Using homological perturbation theory to formalize a finite-dimensional incarnation of Batalin-

Vilkovisky path integrals, we introduce a categorical perspective on quantum L-infinity algebras generalizing the minimal model theorem.

Alexander Zuevsky: *Canonical extensions of vertex algebra bundles*
The purpose of this talk is to construct and determine properties of natural extensions of vertex operator algebra bundles. We describe transformations that preserve vertex operator algebra bundles convergence and canonicity properties. Applications follow in cohomology theory of vertex algebras and underlying manifolds.

GENERAL INFORMATION

44th WINTER SCHOOL GEOMETRY AND PHYSICS
Srní, Czech Republic
January 13–20, 2024

https://conference.math.muni.cz/srni/

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