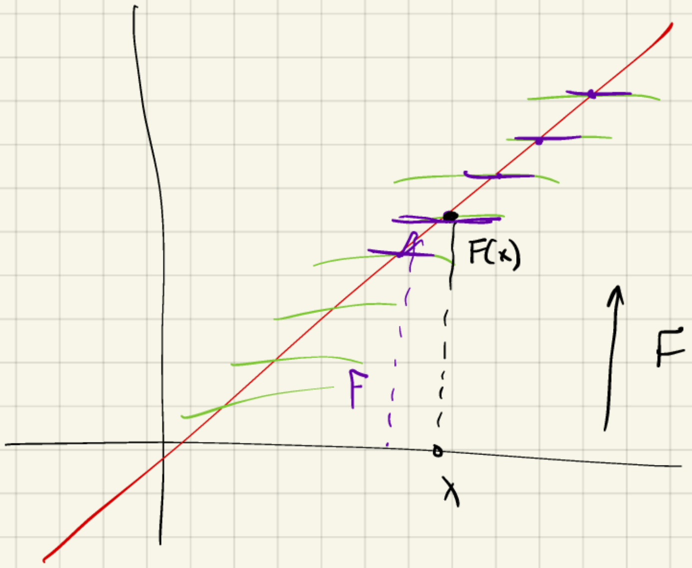


$$J^1(\mathbb{R},\mathbb{R}) \cong \mathbb{R}^3 = \{(x,y,z)\}$$ slope

$$\cup$$

$$R = \{ z=0 \}$$

$$j^1_x F = F(x)$$

Main problem

Fix part. diff. rel $\quad R \subset J^r E$

$$\begin{array}{c} \downarrow \\ E \longrightarrow M \end{array}$$

- Is the scanning map

$$\mathrm{Sols}_R(M) \hookrightarrow \mathrm{Sols}^f_R(M) = \Gamma(R)$$

a weak hom. eq ?

- π_0 - surjectivity

Given F formal $\overset{\text{hom}}{\underset{?}{\rightsquigarrow}}$ $f \in \mathrm{Sol}$.

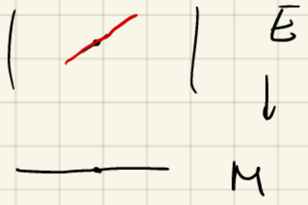
II. Flexible sheaves (Gromov, late 60's)

Idea: Cover M by little balls $\{ U_i \}$

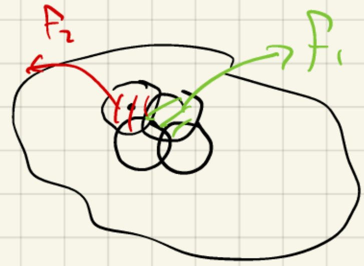
$$F \in \mathrm{Sols}^f_R(M) \underset{\text{local integrability}}{\overset{\text{I}}{\rightsquigarrow}} \{ f_i \in \mathrm{Sols}_R(U_i) \} \underset{\text{Flexible sheaves}}{\overset{\text{II}}{\rightsquigarrow}} f \in \mathrm{Sols}_R(M)$$

(I) In general, not doable

We focus on R open



$\Rightarrow$ any local extension is a solution in a small ball.



(II) $Sols_R$ is a sheaf

$$\begin{array}{ccc} \mathcal{U} & \longmapsto & Sols_R(\mathcal{U}) \\ \cap & & \cap \\ \mathcal{O}(M) & \longrightarrow & Top \end{array}$$

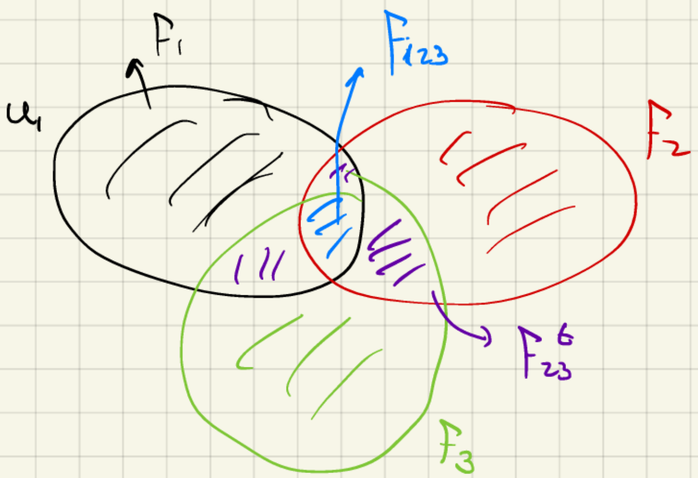
$\{F_i \text{ on } \mathcal{U}_i\}$

agreeing in overlaps glue to a global F.

<u>Claim</u>: we have more than $\{F_i \text{ in } \mathcal{U}_i\}$,

we also have homotopy compatibilities $F_{ijk\ldots}$ in $\mathcal{U}_i \cap \mathcal{U}_j \cap \mathcal{U}_k \ldots$

$j^r F_1$, $j^r F_2$ are close to F and we have interpolation $(F_{12}^t)_{t \in [0,1]}$



$(F_{123}^t)_{t \in \Delta_2}$

Rephrasing of main question:

- is $Sols_R$ a homotopy sheaf ?

  i.e can the collection

  $\{F_I$ in $U_I\}$ which matches up to homotopy,

  glue up to homotopy to $f \in Sol_R(M)$?

- Exercise: $Sols_R^f$ are a homotopy sheaf

  (true for sections of any mapping)

Thm Fix • $F: M \to R \subset J^rE$

• $\varepsilon : M \to \mathbb{R}^+$

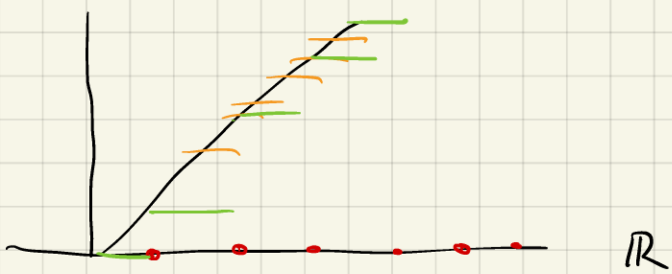
disjoint codim-1 discs

Then there is

$$f: M \setminus \coprod B_i \longrightarrow E \quad \text{s.t}$$

$$j^r f \underset{\varepsilon}{\sim} F$$

- Obs: $\dim(M) = 1$ uninteresting



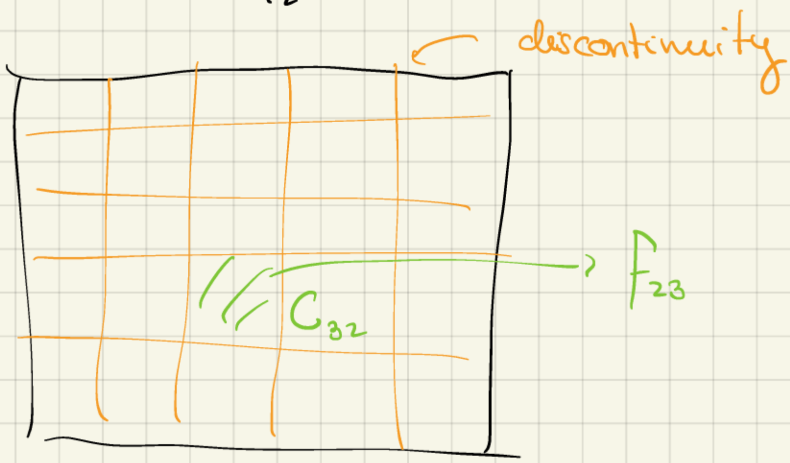
- Corollary: If R open then $\exists\ \varepsilon$ s.t

every F w/ $j^rF \underset{\varepsilon}{\sim} F \in Sol^f_R(M)$ is a solution.

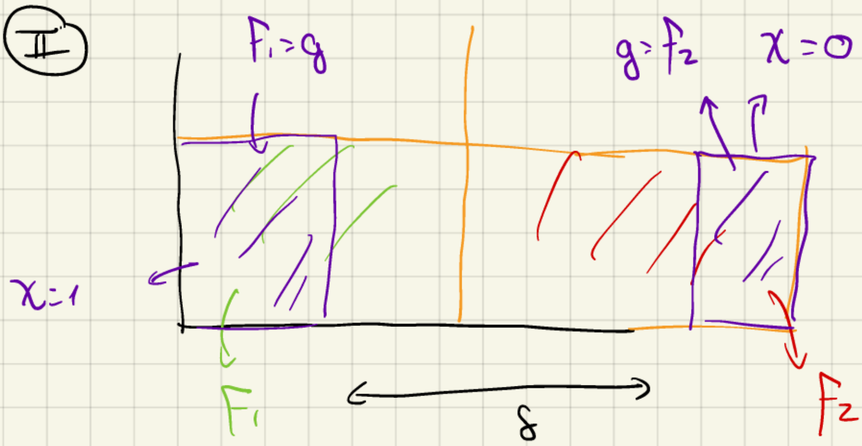
$\Rightarrow$ we construct sols in $M \setminus \coprod B_i$

PF / ∘ We can assume $M = [0,1)^n$ working by charts.

(I) Pick δ small so that M is divided into δ-cubes C_I and in each we have $f_I : C_I \to E$ which is $\varepsilon/2$ - close to F.



(II)



look at

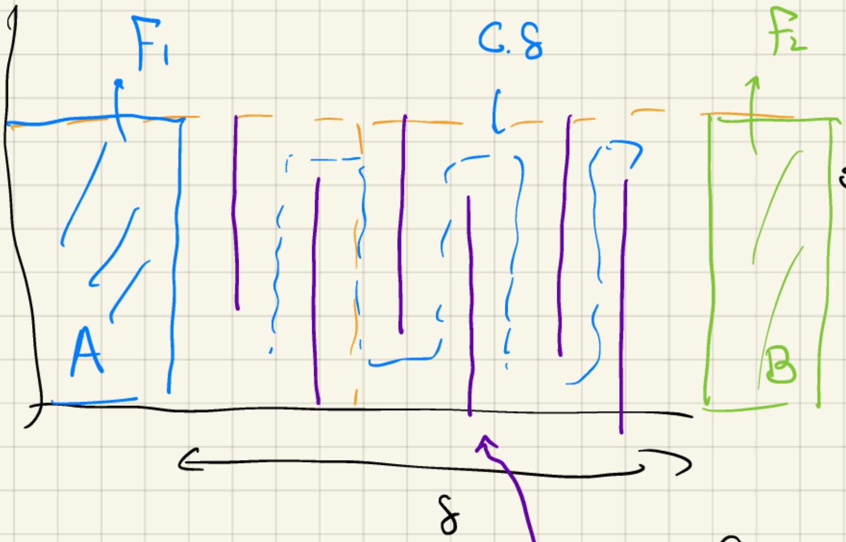
$g = f_1 . x + f_2(1-x)$

x bump from 0 to 1

- at zero order $g \underset{C^0}{\simeq} F_1$ everywhere
- but

$$Dg = \overbrace{\left[Df_1 . x + Df_2(1-x) \right]}^{\text{great}} + \underbrace{\underbrace{dx}_{1/\delta} . \underbrace{(F_1 - F_2)}_{\delta}}_{O(1)}$$

Introduce C cuts so distance from



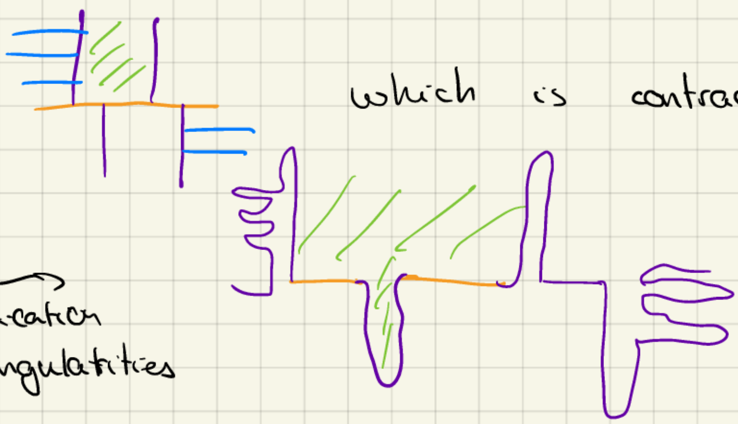
So now we have $\chi \begin{cases} 1 \text{ in } A \\ 0 \text{ in } B \\ \text{of arbitrarily small derivative } C \to \infty \end{cases}$

! discontinuous

We produce section g w/ discontinuities along [purple] and [orange] but $j^r g \sim F$

Do same vertically.

Q: how bad are these



which is contractible

"wiggly disc"

simplification of singularities

$\square$

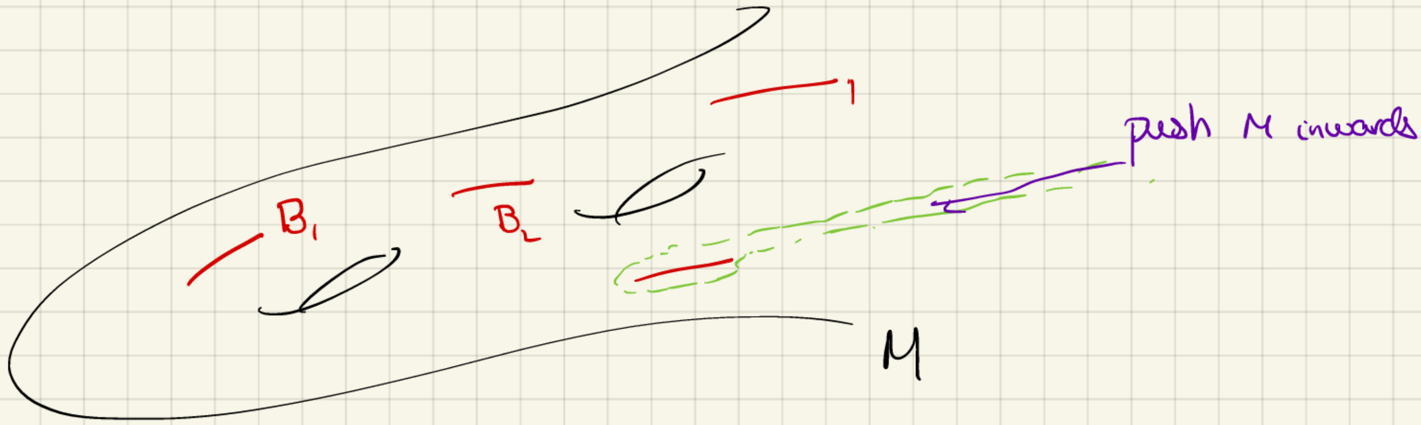
Corollary (Gromov's h-principle in open manifolds)

Let M open (every component not closed)

$\mathcal{R}$ open + Diff - invariant

$\Rightarrow$ $\mathcal{R}$ satisfies h-principle.

Pf/ (out of formal sol $F \rightsquigarrow f \in \mathrm{Sols}_{\mathcal{R}}(M)$)



M open $\Rightarrow$ $\exists$ $\varphi : M \hookrightarrow M \setminus \amalg B_i$ embedding.

By thm we had $f : M \setminus \amalg B_i \longrightarrow E$ solution

by diff-inv $\varphi^* f \in \mathrm{Sols}_{\mathcal{R}}(M)$ $\square$

Q: · what about M closed?

· M open but solutions controlled at ∞?

$\mathrm{Imm} \simeq \mathrm{Mon}$ (Smale - Hirsch)