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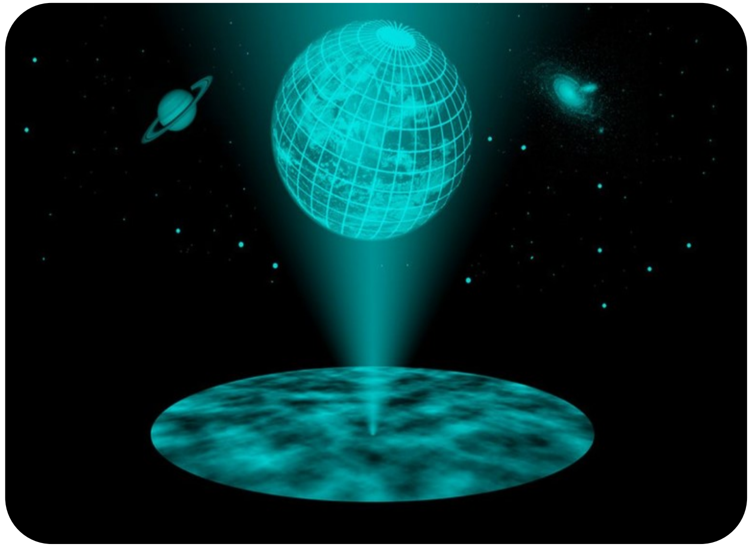
Categorification of the holographic principle



44th Winter School Geometry and Physics,

Srní, 15th of January, 2024

Petr Vlachopulos, Department of Mathematics and Statistics, Masaryk University

- **Holographic principle ('t Hooft):** Given any closed surface, we can represent all that happens inside it by degrees of freedom on this surface itsel.
- Bekenstein area law

$$S = \frac{A}{4}$$

→ Number of degrees of freedom cannot exceed $A/4$.

- **Holographic principle (Susskind):** Explicit mapping from volume to surface degrees of freedom for a general closed system.
- All light rays that are normal to any element of the surface within the bulk are also normal to the boundary
- Covariant entropy bound:

$$S\big(L(\Sigma)\big) \leq \frac{A(\Sigma)}{4}$$

  - „Quantum" only in its reliance on Planck units, finite value of Planck constant
  - S – thermodynamical entropy

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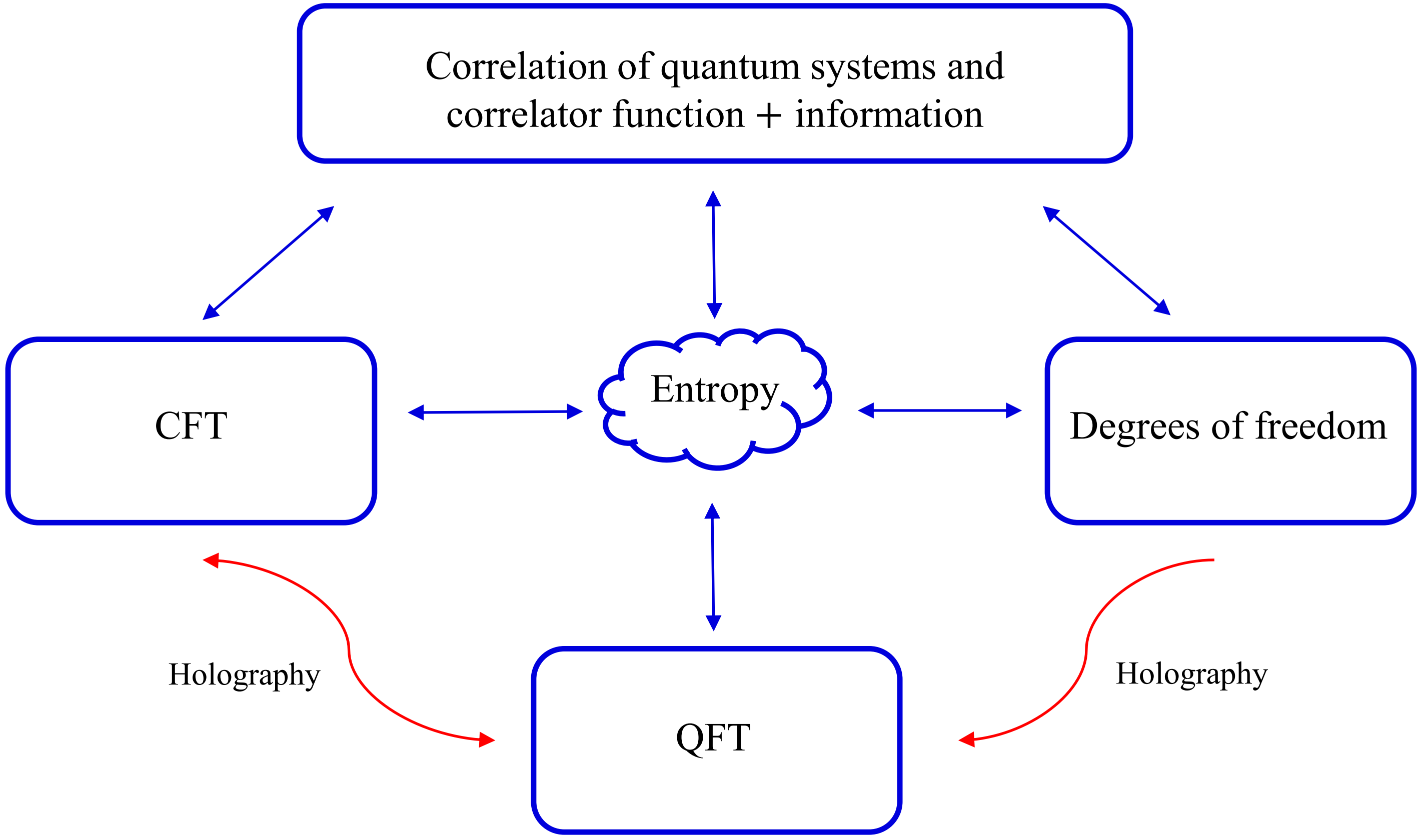
AdS-CFT duality, entanglement holographic entropy and physical meaning of holography



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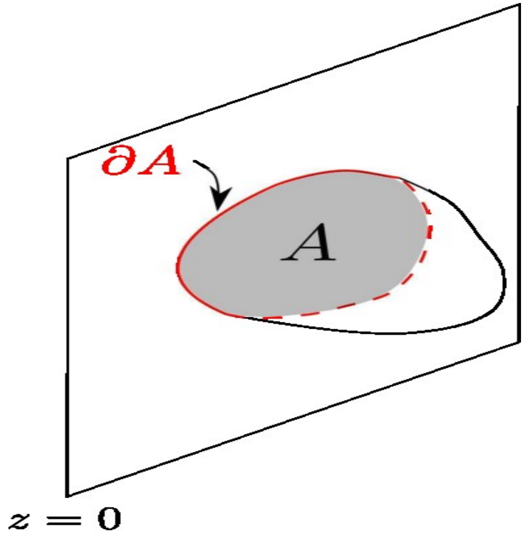
Entanglement entropy (EE). Measures how two quantum states A and B are entangled (correlated).

$$S_A = Tr[\rho_A log(\rho_A)]$$

- In 2-dim. CFT, EE is proportional to central charge (it's easy to compute)
- In $d > 2$, it's not easy to compute EE for arbitrary submanifold A

- Holographic estimation of EE by using the AdS-CFT duality

- $(AdS)_{d+2}$ is holographically related to $(CFT)_{d+1}$
- Bekenstein-Hawking correspondence for entropy (Black holes)



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•**Maldacena:** String QG on a bulk $(AdS)_{d+2}$ and a CFT on its $(d+1)$ – dimensional boundary encode the same information

… an apparent law of physics that stands by itself, both uncontradicted and unexplained by existing theories, that may still prove incorrect or merely accidental, signifying no deeper origin

•Implementation of maximal classical information flow within physical interaction between mutually separable systems

Let $\mathcal{U} = AB$ is a finite closed system. Then if $|AB\rangle = |A\rangle|B\rangle$ (separable), the classical information exchange between A and B is limited to N bits, which corresponds to the dimension of the interaction Hamiltonian H_{AB}.

Information lies on the boundary between A and B.

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Cons

- Heuristic proces
- No axiomatics
- Nonexisting rigorous mathematical framework

Pros

- Straightforward
- Physically meaningful
- Captures the notion of von Neuman entropy by holographic screens and minimal hypersurfaces in the bulk spacetime

Formalization of holography ?

Possible approach

- Categorification
- Reflective subcategories
- Reflective localization

Reflective subcategories

- full subcategory i $: X \hookrightarrow Y$ is reflective if the inclusion functor i has a left adjoint $T : Y \to X$
- for objects y and morphisms $f : y \to y'$ in Y we have reflections TY and $Tf : Ty \to Ty'$ in X
- T – reflector, i – reflection, object in Y looks on its reflection via $y \to Ty$

Examples:

- $Ab \hookrightarrow Grp$, reflector – abelianization
- Compact Hausdorff spaces – reflective subcategory of $TopSpace$, reflector – Stone-Čech compactification
- Category of Banach spaces – reflective subcategory of $NormSpace$**,** reflector – norm completion functor

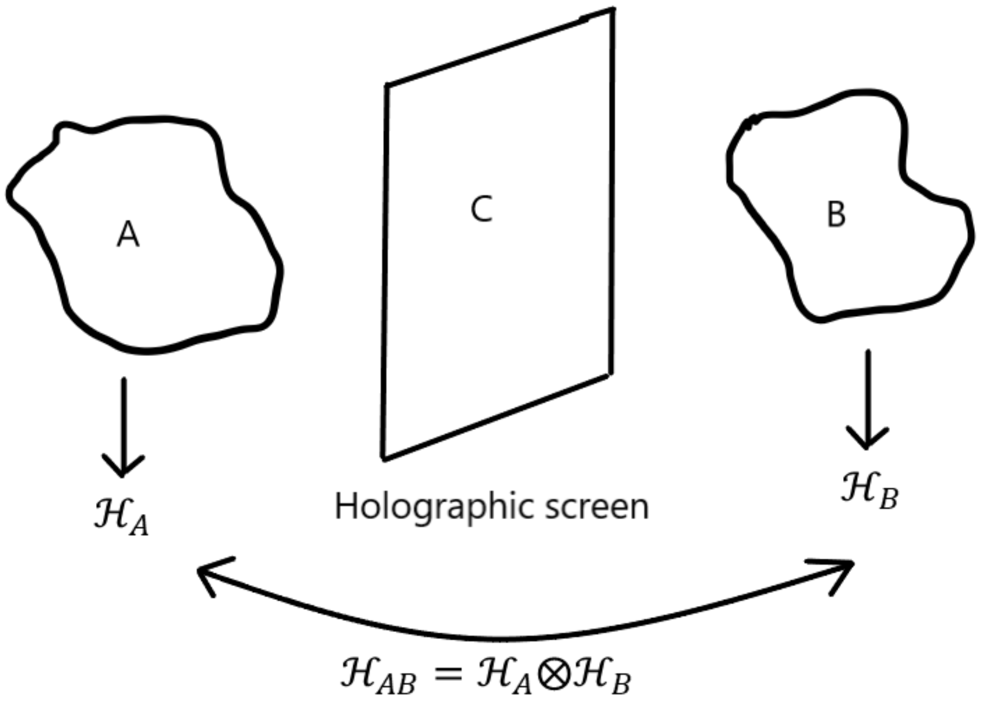
Prop.

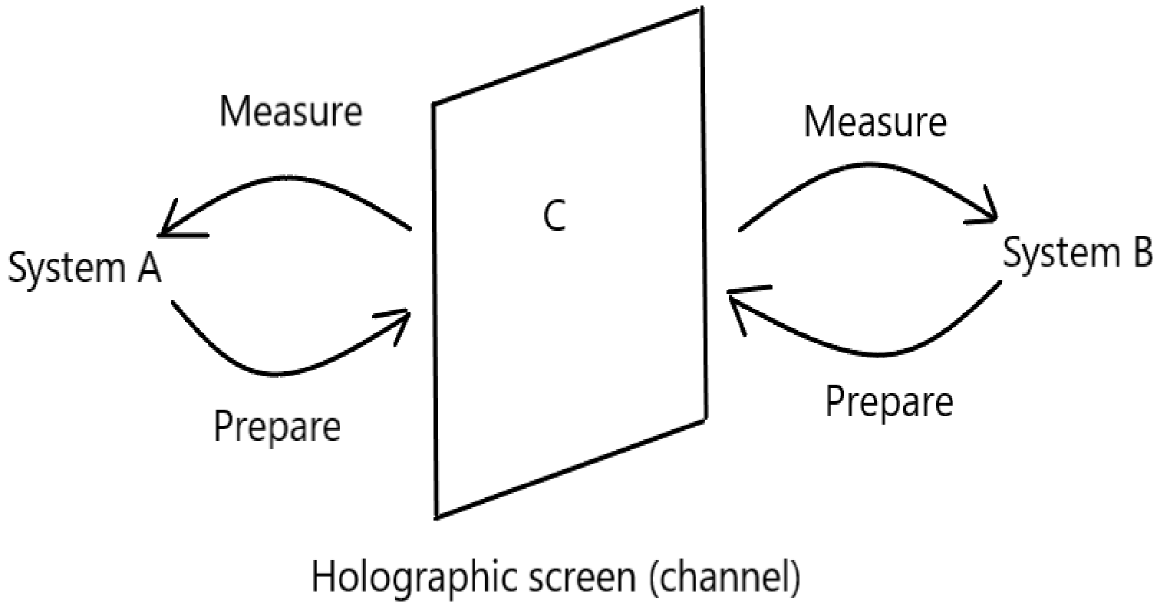
Given any pair of adjoint functors $\Psi : B \to A$, $\Phi : A \to B$, the following are equivalent

1) The right adjoint Ψ is fully faithful - B is full reflective subcategory of A
2) The counit $\varepsilon : \Phi\Psi \to Id_B$ of the adjunction is a natural isomorphism of functors
3) If M is a set of morphisms m in A such that $\Phi\,(m)$ is an iso in B, then $\Phi : A \to B$ realizes B as localization of A in relation to M

General properties

1) Reflective subcategory is closed under limits – inclusion functor creates all limits
2) Reflective subcategory has all colimits that are present in the ambient category





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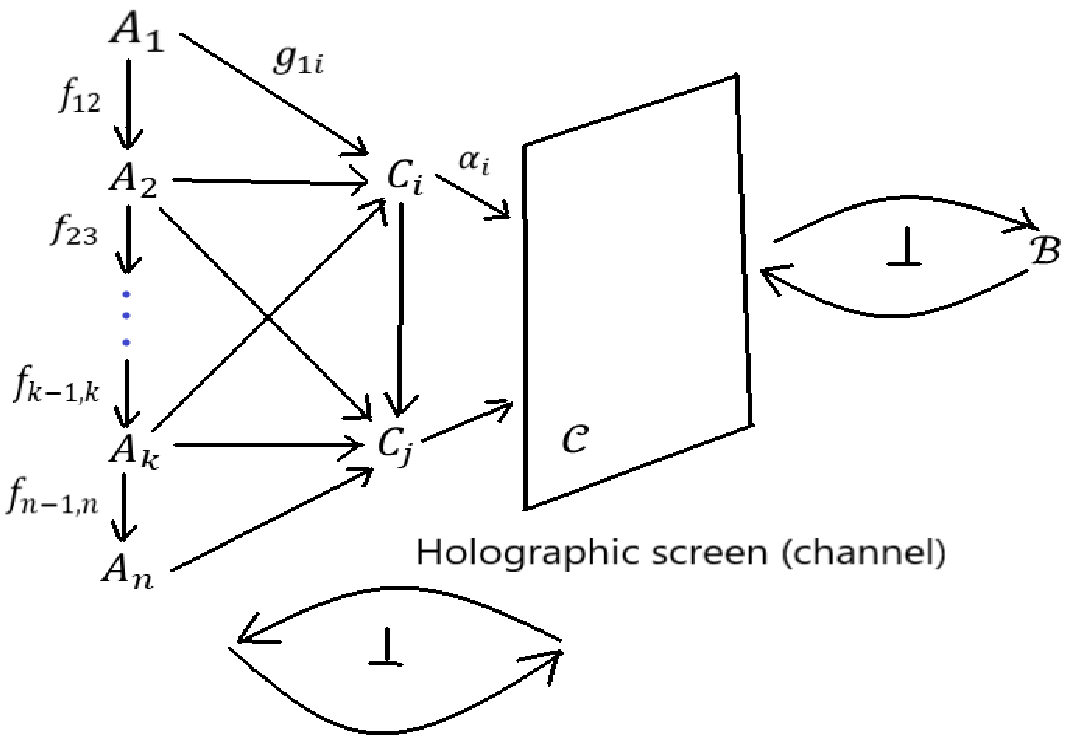
Categorification

- $\mathcal{C}$ – information channel contains all the constraints jointly encoded by objects A_i in A
- $\mathcal{C}$ functions as a shared memory storage, jointly accessed by objects and morphisms $f_{ij} : A_i \to A_j$ in A
- There exist morphisms $g_{ij} : A_i \to C_j$ that maps from objects in A to one or more information channels (objects of holographic screen) C_i
- There exist $\alpha_i : C_i \to \mathcal{C}$ from information channels to $\mathcal{C}$ („colimit“)



Reflective localization of measurements with respect to $\mathcal{C}$

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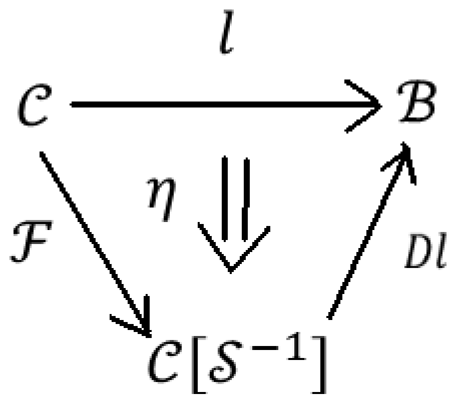
Reflective localization of measurements

- Use category with weak equivalences – we need isomorphisms to describe identical states
- Category $\mathcal{C}$ and subcategory $\mathcal{S}$ of $\mathcal{C}$ such that
  - ❑ $\mathcal{S}$ include all isomorphisms of $\mathcal{C}$
  - ❑ if two out of three composable morphisms g, f, gf are in $\mathcal{S}$, then so is the third one

Localization of a category

- $\mathcal{S} \subset \mathcal{C}$ category with weak equivalences, then the localization of $\mathcal{C}$ at $\mathcal{S}$ corresponds to
  - ❑ category $\mathcal{C}[\mathcal{S}^{-1}]$
  - ❑ functor $\mathcal{F} : \mathcal{C} \to \mathcal{C}[\mathcal{S}^{-1}]$ (sends all morphisms in $\mathcal{S}$ to isomorphisms) and $\mathcal{F}$ is universal with this property

- Localization is reflective if the localization functor has a fully faithful right adjoint – reflective subcategory inclusion



Every reflective subcategory inclusion is the reflective localization with respect to the class of morphisms that are sent to isomorphisms by the reflector

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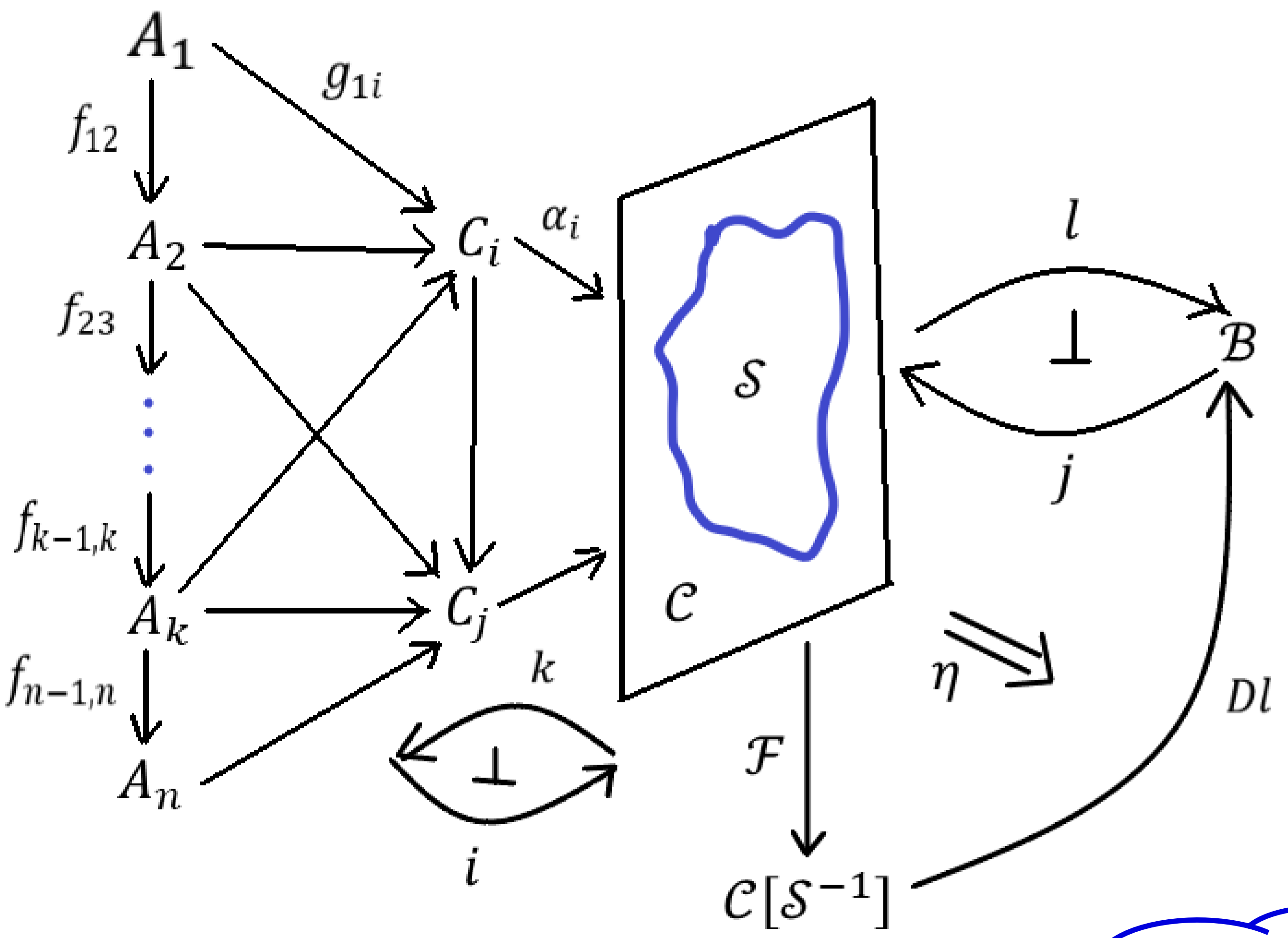


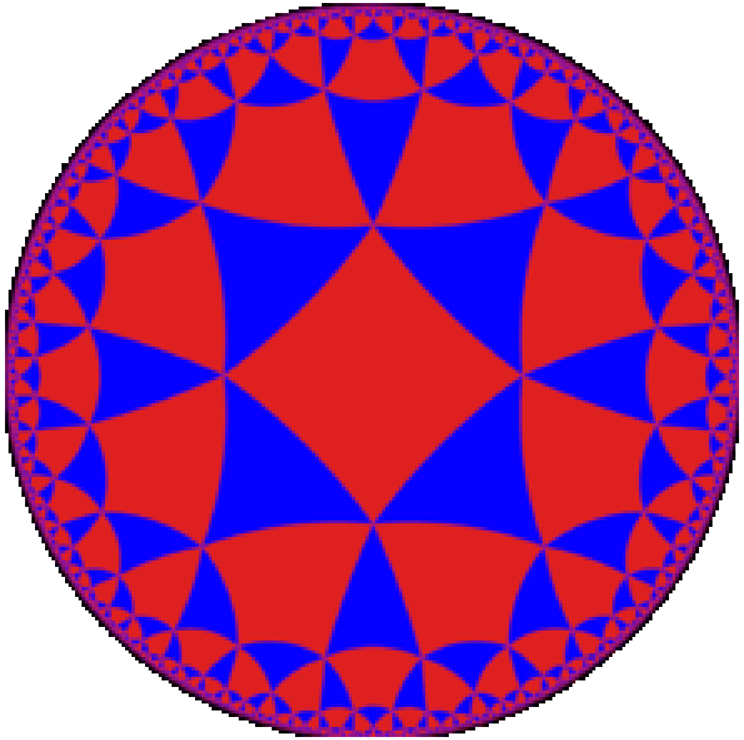
- ***Preparation*** – inclusion (i) of the full subcategory onto the $\mathcal{S}$-objects
- ***Measurement*** – reflection (k) of the full subcategory onto $\mathcal{S}$-objects

Holographic principle can be encoded by reflective localization of categories ?

Higher holographic duality

?

Reflective localization of higher categories



Thank you for your attention