# THE 45$^{\text{rd}}$ WINTER SCHOOL
## GEOMETRY AND PHYSICS

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Sponsored by

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# ANNOUNCED LECTURES

## A. Invited lectures

**Gil Cavalcanti**: *T-duality: from circles to spheres and beyond*

**Olivier Guichard**: *The geometry of flags manifolds*

**Sylvie Paycha**: *A prolegomenon to renormalisation in three movements: or a (desperate?) attempt to make the infinite finite*

**Urs Schreiber**: *Introduction to Hypothesis H*

**Bruno Vallette**: *An operadic approach to deformation theory*

**Vera Vertesi**: *Giroux correspondence via convex surfaces*

## B. Other lectures

**Teresa Arias-Marco**: *The Laplace-Beltrami operator and the symmetric-like properties on Riemannian manifolds*

**Hannes Berkenhagen**: *The spinor bundle on loop space*

**Arnab Bhattacharjee**: *Bimodule connection for relative Hopf module over irreducible quantum flag manifold*

**Samuel Blitz**: *Conformal Constraints on Asymptotically Kerr-de Sitter*

**Eugenia Boffo**: *The B-model field theory in BV formalism*

**Sebastian Brezina**: *Principal bundles, gauge theory, and gravity: Teleparallel Equivalent of General Relativity*

**Benedek Bukor**: *Cubic asymmetric multitrace matrix model*

**Andreas Čap**: *Chern-Simons invariants and flat extensions*

**Alessandro Carotenuto**: *Convex orderings and quantum tangent spaces*

**Carlo Alberto Cremonini**: *An alternative to Homotopy Transfer*

**Antonio Del Donno**: *Principal Bundles and Differential Structures in Noncommutative Geometry*

**Zdeněk Dušek**: *Structure of geodesics for Finsler metrics arising from Riemannian g.o. metrics*

**Matthias Frerichs**: *Coarse geometric approach to topological phases of matter*

**Gianni Gagliardo**: *Adjusted Connections for Principal 3-Bundles*

**Anton Galaev**: *Conformally homogeneous Lorentzian spaces*

**Alejandro García Rivas**: *Exotically knotted surfaces*

**Roman Golovko**: *Lagrangian concordance is not a partial order*

**Jan Gregorovič**: *Irreducible (conformal) Killing tensors on homogeneous plane waves*

**Leszek Hadasz**: *Airy structures and supergeometry.*

**Stepan Hudeček**: *Poisson equation for $G_2$-Laplacian on homogeneous spheres*

**Ondrej Hulík**: *Generalized Symmetries, TFT and T duality*

**Goce Chadzitaskos**: *On the Mach's principle.*

**Ioannis Chrysikos**: *Curvature of quaternionic skew-Hermitian manifolds and bundle constructions*

**Josef Janyška**: *The 1st Maxwell's equation over the phase space of spacetime*

**Branislav Jurčo**: *Lagrangian Relations and Quantum L-infinity Algebras*

**Igor Khavkine**: *Simplifying harmonic gauge perturbations around black holes*

**Jakub Knesel**: *Representation Theory, Schubert Calculus and Algebraic Topology of Some Embedded Flag Subanifolds*

**Daniel Komárek**: *On quantitative aspects of symplectic geometry*

**Andrey Krutov**: *SuperLie: a package for Lie (super) computations*

**Svatopluk Krýsl**: *Primitive forms for symplectic twistor operators*

**Tim Lüders**: *Graded-equivariant bundle gerbes and twisted K-theory*

**Marián Lukáč**: *Courant Algebroids*

**Antonella Marchesiello**: *Superintegrable families of magnetic monopoles in curved background*

**Filip Moučka**: *Cn-generalized complex geometry*

**Jiří Nárožný**: *One-jets of groupoids – bisection construction approach*

**Katharina Neusser**: *Cone structures and conic connections*

**Reamonn Ó Buachalla**: *Quantum exterior algebras and torsion free bimodule connections*

**Pavle Pandzic**: $SL(2)$ *theory and quantum computing*

**Tomáš Procházka**: *Harmonic oscillator, 2d free boson and integrability*

**Ľuboš Ravas**: *BRST and beyond*

**Andrea Rivezzi**: *On the universal Drinfeld-Yetter algebra*

**Martin Schnabl**: *On classical solutions in open string field theory*

**Libor Šnobl**: *On the incompleteness of the classification of quadratically integrable Hamiltonian systems in the three-dimensional Euclidean space*

**Rudolf Šmolka**: *Elements of Graded Lie Theory*

**Matouš Trnka**: *Projective invariants of non-torsal ruled surfaces*

**Dominik Trnka**: *Unstraightening of an operad*

**Fridrich Valach**: *On full supergravity*

**Vladimír Souček**: *The Jakobsen diagrams and unitary highest weight modules*

**Jan Vysoký**: *Graded Principal Bundles and Connections*

**Thomas Weber**: *The crossed product calculus*

**Henrik Winther**: *Higher order connections in noncommutative geometry*

**Lenka Zalabová**: *Conserved quantities for conformal loxodromes on conformal sphere*

**Martin Zika**: *Spans of Quantum L-infinity Algebras*

**Alexander Zuevsky**: *Applications of CFT determinant formulas in number theory*

## C. Posters

**Chen-Hsu Chien**: *Scalar Field Embedding in Cartan geometry for Gauge Theory of Gravity*

**Branislav Jurčo**: *Lagrangian Relations and Quantum L-infinity Algebras*

**Lukáš Krump**: *Selfdual 2–forms in dimension 4 and their Fischer decomposition*

**Mauro Mantegazza**: *Jets in Noncommutative Geometry*

**Petr Vlachopulos**: *Advancements in the Yau's conjecture on the first eigenvalue*

# ABSTRACTS

**Teresa Arias-Marco**: *The Laplace-Beltrami operator and the symmetric-like properties on Riemannian manifolds*
The main objective of the talk is to present the latest advances made in the determination of a symmetric-like property using the Laplace-Beltrami operator on Riemannian manifolds. Symmetric-like properties are those which generalize the local symmetry on Riemannian manifolds. In this context, we focus in the existence of an operator which intertwines the Laplace-Beltrami operator of two Riemannian manifolds with certain symmetric-like properties. Joint work with José Manuel Fernández-Barroso.

**Hannes Berkenhagen**: *The spinor bundle on loop space*
Given the connection of spin geometry to particle physics,string geometry tries to find analogous structures with uses in string theory. One way to tackle this is to think of closed strings in $M$ as points in the loop space $LM$ and to use ideas from the framework of point-like particles moving through a manifold, though here differences to the finite-dimensional case arise as $LM$ is an infinite-dimensional Fréchet manifold. The talk will center around describing analogues of spin geometric structures on the loop space – namely loop spin structures and the spinor bundle on $LM$. I will then explain ongoing work for my PhD thesis under Konrad Waldorf, where we try to follow works of Brylinski and Witten in order to extract elliptic cohomology classes from these loop spinor bundles.

**Arnab Bhattacharjee**: *Bimodule connection for relative Hopf module over irreducible quantum flag manifold*
In 2020, in the paper Holomorphic Relative Hopf Modules over Irreducible Quantum Flag Manifolds, the authors showed that there exists unique covariant connection for relative Hopf modules over irreducible quantum flag manifolds. In this talk, we shall extend this result and show that for every relative Hopf modules over irreducible quantum flag manifolds, there always exists a covariant bimodule connection.

**Samuel Blitz**: *Conformal Constraints on Asymptotically Kerr-de Sitter*
Kerr-de Sitter spacetimes are physically interesting, as they form a general family of physical black holes. It has been shown by Frolov, Krtouš, and Kubizňák that if a spacetime with positive cosmological constant admits a principal conformal Killing-Yano 2-form, then the spacetime is a Kerr-NUT-de Sitter spacetime, which is a generalization of the Kerr-de Sitter spacetime. Using conformal methods, we construct a series of constraint equations on the conformal infinity of a spacetime to admit such a tensor, and hence are necessary conditions for the spacetime to be asymptotically Kerr-de Sitter.

**Eugenia Boffo**: *The B-model field theory in BV formalism*
The effective field theory of the $B$-model of topological string theory was first proposed by Bershadsky–Cecotti–Ooguri–Vafa and describes deformations of complex structures. Later on, Costello–Li ventured in the formalisation and generalisation of this theory. However a BV action functional is still missing, because the classical action involves non-local differential operators. We wish to fill this gap by suggesting a local BV action functional, constructed using a trick due to Sen. Based on joint on-going work with Ivo Sachs.

**Sebastian Brezina**: *Principal bundles, gauge theory, and gravity: Teleparallel Equivalent of*

*General Relativity*
Folklore holds that the principal bundle formalism provides a rigorous and effective framework for describing classical gauge theories of internal symmetries, such as Yang-Mills theories and electromagnetism. However, it is not well known that this formalism also allows one to describe gauge gravitational theories completely, in contrast to standard claims that the gauge approach to gravity requires some special treatment compared to the gauge theories of internal symmetries. Trautman showed all of this during the 1970s by precisely defining gauge transformations of a gauge theory containing non-dynamical elements. The canonical 1-form living on the frame bundle is considered a non-dynamical element of gravitational theories based on dynamical metric tensor and linear connection. This talk explains how bundle formalism naturally incorporates gauge gravitational theories and focuses on whether the Teleparallel Equivalent of General Relativity is a "gauge theory of translations"as it is often presented.

**Benedek Bukor**: *Cubic asymmetric multitrace matrix model*
We analyze multitrace random matrix models with the help of the saddle point approximation and we introduce a multitrace term of type $-c1c3$ to the action. We obtain the numerical phase diagram of the model, with a stable asymmetric phase and the triple point. Furthermore, we examine response functions in this model. The talk will be based on joint work with Juraj Tekel, arXiv:2407.20014.

**Andreas Cap**: *Chern-Simons invariants and flat extensions*
The classical Chern-Simons invariant associates to a closed, oriented Riemannian 3-manifold $(M, g)$ an element of $\mathbb{R}/\mathbb{Z}$. This turns out to be conformally invariant and it vanishes if there is a global isometric immersion of $(M, g)$ into $\mathbb{R}^4$. A similar construction leads to the real-valued Burns-Epstein invariant of a compact oriented 3-dimensional CR manifold that admits a global CR vector field.
The talk reports on joint work with K. Flood and T. Mettler in which we first define analogous global invariants for certain types of connection forms on principal bundles over compact oriented 3-manifolds. Second, in the case of principal connections, we develop a concept of flat extension which implies vanishing or integrality of the Chern-Simons invariant. Applications to Lorentzian metrics, affine connections, and to parabolic contact structures on 3-manifolds are discussed.

**Alessandro Carotenuto**: *Convex orderings and quantum tangent spaces*
When building up a theory of differential noncommutative geometry, one of the most delicate steps is the construction of a suitable differential calculus that describes the differential structure of a given noncommutative space. In a recent work Ó Buachalla and Somberg showed a covariant differential calculus of classical dimension for the quantum full flag manifolds of type $A$. They made use of the celebrated theory of Lusztig bases for the quantized enveloping algebras to build up a so-called quantum tangent space. In order to extend this result to other series, and hopefully prove the uniqueness of such differential calculi, one has to find a unifying framework to describe the coproduct of Lusztig root vectors. In this talk, I will show a way to do it in terms of convex orderings on positive roots, pointing at the combinatorial properties of a reduced decomposition of the longest element of the Weyl group that gives rise to a quantum tangent space.

**Gil Cavalcanti**: *T-duality: from circles to spheres and beyond*
In this three-lecture series we will review from a mathematical perspective First we will cover the mathematics of T-duality for $S^1$ symmetries as initially introduced by Buscher in the 80's and further developed mathematically in the 00's. In the second lecture we will see how to extend

T-duality to sphere bundles as introduced in the 10's and see what new features emerge in this situation. In the last lecture we will further extend the concept of T-duality to transgressive fibrations, as done in the second half of the 20's ;-)

**Carlo Alberto Cremonini**: *An alternative to Homotopy Transfer*
In this talk, I will discuss the draft arXiv:2406.12508. Here we develop a construction inspired by String Field Theory to induce an $A_\infty$ structure on a complex starting from algebraic data on a homotopic equivalent complex, where the two are equipped with some extra structure. Under some simplifying assumptions this procedure is quasi-isomorphic to the one given by the usual homotopy transfer theorem, under milder assumptions the two methods are not quasi-isomorphic.

**Antonio Del Donno**: *Principal Bundles and Differential Structures in Noncommutative Geometry*
This talk reviews the concepts of principal bundles and differential calculi within noncommutative geometry. We adopt a purely algebraic perspective where Hopf algebras and comodule algebras replace topological spaces. Building on the work of Durdevic, we discuss complete differential calculi, the Atiyah sequence, and a canonical braiding operation, considering the quantum Hopf fibration as fundamental example. The presentation is based on joint work with Emanuele Latini and Thomas Weber.

**Zdeněk Dušek**: *Structure of geodesics for Finsler metrics arising from Riemannian g.o. metrics*
Homogeneous geodesics of homogeneous Finsler metrics derived from two or more Riemannian geodesic orbit metrics are investigated. For a broad newly defined family of positively related Riemannian geodesic orbit metrics, a geodesic lemma is proved and it is shown that the derived Finsler metrics have also geodesic orbit property. These Finsler metrics belong to the newly defined class of the $\alpha_i$-type metrics which includes in particular the $(\alpha_1, \alpha_2)$ metrics.
An example of a sphere $S^7$ with geodesic orbit Finsler metrics of the new type $(\alpha_1, \alpha_2, \alpha_3)$, arising from two or more Riemannian geodesic orbit metrics, is analyzed in detail.
Joint work with Teresa Arias Marco.

**Matthias Frerichs**: *Coarse geometric approach to topological phases of matter*
Invariants of topological phases of matter used in solid state physics are often extracted by considering topological invariants of the Bloch-bundle, for example via its Chern number in dimension 2 to determine Chern insulators. This, however, conceptually depends on the periodicity of the underlying lattice of atoms, which in reality can not be guaranteed and is completely absent in amorphous materials.
I give a short introduction to determining such invariants directly from the Hamiltonian by looking at the $K$-theory an associated Roe-Algebra. In combination with Coarse cohomology classes of the underlying set of atoms, this should replicate the classical topological invariants determined via the Bloch bundle.

**Gianni Gagliardo**: *Adjusted Connections for Principal 3-Bundles*
Higher-form gauge fields arise naturally in physics in various contexts, such as the Kalb-Ramond field in sting theory or the $C$-field in M-theory and supergravity. However, the study of global aspects of theories with non-abelian higher-form gauge fields has faced limitations due to technical challenges. Specifically, to ensure consistency when gluing gauge transformations for non-abelian gerbes one is forced to insist that the fake curvature vanishes. In this talk, we shall describe a way to circumvent this problem by introducing the notion of adjustments for 2-crossed modules. This enables us to give a complete cocycle description of non-abelian principal 3-bundles with adjusted connections, which we prove can be glued consistently without the fake

flatness requirement. Furthermore, we present possible applications to the study of U-duality, inspired by recent work studying T-duality using higher gauge theory by Nikolaus and Waldorf (2019), and Kim and Sämann (2022).

**Anton Galaev**: *Conformally homogeneous Lorentzian spaces*
This is a joint work with Dmitri Alekseevsky. We prove that if a simply connected non-conformally flat conformal Lorentzian manifold $(M, c)$ admits an essential transitive group of conformal transformations, then there exists a metric $g \in c$ such that $(M, g)$ is a complete homogeneous plane wave. We also prove that the group of conformal transformations of a non-conformally flat simply connected homogeneous plane wave $(M, g)$ consists of homotheties, hence it is a 1-dimensional extension of the group of isometries.

**Alejandro García Rivas**: *Exotically knotted surfaces*
We give several examples of exotic embeddings of surfaces in 4-manifolds. These are embeddings of surfaces that are isotopic through homeomorphisms but not through diffeomorphisms. In particular, we extend Kyle Hayden's construction [1] to obtain arbitrarily large tuples of exotic embeddings of any surface with boundary in the 4-ball (relative boundary). We also show that we are able to remove the "relative boundary"restriction in some cases. In order to explain this construction, a detour through Kirby calculus, contact geometry and Stein structures is needed.
[1] Kyle Hayden, Corks, covers and complex curves, 2021

**Roman Golovko**: *Lagrangian concordance is not a partial order*

**Jan Gregorovič**: *Irreducible (conformal) Killing tensors on homogeneous plane waves*
On homogeneous conformal and projective geometries, we can find solutions of BGG operators using algebraic methods. This allows for class of such geometries discuss the number of solutions by investigating ranks of a systems of linear equations. In a joint work with L. Zalabová, we do it for the class of homogeneous plane waves and identify the metrics in this class that have irreducible Killing and conformal Killing tensors.

**Olivier Guichard**: *The geometry of flags manifolds*
TBA

**Leszek Hadasz**: *Airy structures and supergeometry.*
During my presentation I will show how Airy structures – an algebraic version and generalization of topological recursion – paves a way to construct a supersymmetric version of Konstsevich matrix integral and offers a new viewpoint on moduli space of super-Riemann surfaces.

**Ondřej Hulík**: *Generalized Symmetries, TFT and T-duality*

**Goce Chadzitaskos**: *On the Mach's principle.*
We derived a relation for Mach's principle by applying Schwarzschild's procedure to solve Einstein's equations within a homogeneous mass sphere. We then use this conclusion to extend the solution to a three–dimensional sphere as a model of the spatial component of space–time. For a sta- tionary model of the universe as a three–dimensional sphere in four– dimensional space, and the corresponding possible proportionality coefficients are calculated.

**Chen-Hsu Chien**: *Scalar Field Embedding in Cartan geometry for Gauge Theory of Gravity*
Scalar fields play a crucial role in cosmology, serving as potential candidates for dark energy or as inflaton fields that control the evolution of the universe. Given that gravity can be described using Cartan geometry, it would be fascinating to explore whether scalar fields can also arise

from this mathematical framework. Future research could extend this idea to Yang-Mills theory, potentially incorporating the Higgs field and Dirac field into this geometric perspective.

**Ioannis Chrysikos**: *Curvature of quaternionic skew-Hermitian manifolds and bundle constructions*
In this talk we will describe a curvature characterization of quaternionic skew-Hermitian structures, based on the holonomy theory of symplectic connections. We will also discuss the existence of almost hypercomplex skew-Hermitian structures on the Swann bundle over a quaternionic skew-Hermitian manifold (torsion-free), and describe results on their integrability. This talk is based on a joint work with Vicente Cortés and Jan Gregorovič.

**Josef Janyška**: *The 1st Maxwell's equation over the phase space of spacetime*
The phase space of general relativistic spacetime can be defined as the 1st jet space of motions. A Lorentzian metric and an electromagnetic 2-form allow us to define naturally geometric structures of the phase space. Using these structures we can express the 1st Maxwell's equation via some exterior forms on the phase space. Given an observer, we can obtain, via a pullback, the system of equations on spacetime and prove that this system of equations is equivalent to the system of equations obtained in the standard general relativistic literature.

**Branislav Jurčo**: *Lagrangian Relations and Quantum L-infinity Algebras*
Quantum L-infinity algebras are higher loop generalizations of cyclic L-infinity algebras. Motivated by the problem of defining morphisms between such algebras, we construct a linear category of $(-1)$-shifted symplectic vector spaces and distributional half-densities, originally proposed by Ševera. Morphisms in this category can be given both by formal half-densities and Lagrangian relations; we prove that the composition of such morphisms recovers the construction of homotopy transfer of quantum $L_\infty$-algebras. Finally, using this category, we propose a new notion of a relation between quantum $L_\infty$-algebras.

**Igor Khavkine**: *Simplifying harmonic gauge perturbations around black holes*
I will review a method of simplifying the separated equations of Maxwell and linear gravitational perturbations around a Schwarzschild black hole. The simplified form is a sparse upper triangular coupling of a system of Regge-Wheeler equations. The method takes advantage of a specific notion of equivalence between differential equations and of the special structure of ODEs with rational coefficients. I will also discuss the prospects of applying similar methods to the Kerr black hole.

**Jakub Knesel**: *Representation Theory, Schubert Calculus and Algebraic Topology of Some Embedded Flag Subanifolds*
We provide a representation-theoretical approach to computing Chern classes of normal bundles of embeddings of complex Grassmannians with the use of Schubert calculus, namely the decomposition into Schubert cells and Pieri formula. If time permits, we shall also discuss how the Chern classes of the tangent bundles of complex Grassmannians can be computed only from the knowledge of the Chern classes of the normal bundles, as well as analogous constructions and computations in the real and quaternionic situations with Stiefel-Whitney and Pontryagin classes.

**Daniel Komárek**: *On quantitative aspects of symplectic geometry*
Symplectic manifolds do not posses any local invariants, hence we define some global invariants called symplectic capacities. We briefly introduce ourselves to Embedded contact homology (ECH) and one kind of symplectic capacities – ECH capacities. In adition we present some examples of them.

**Andrey Krutov**: *SuperLie: a package for Lie (super) computations*
SuperLie is a package for Lie (super)algebra computations in Mathematica software system originally developed by Pavle Grozman and currently maintained by A. Krutov. SuperLie is designed for solutions of scientific and computational problems related to Lie (super)algebras, their q-deformations included. Using SuperLie one can construct objects habitual for the mathematician (vector spaces and superspaces, algebras and modules over these algebras) in an explicit way. SuperLie can solve various applied problems and theoretical problems of considerable importance to the physicists. In particular, SuperLie allows one to perform calculations and symbolic transformations in order to determine generators and relation of Lie (super)algebrass, vacuum vectors (highest and lowest), compute Lie (super)algebra homology and cohomology; calculate the Shapovalov determinant, Cartan prolongations and so on. In the talk we will discuss several examples.

**Svatopluk Krýsl**: *Primitive forms for symplectic twistor operators*
On Kaehler manifolds, the notion of primitive forms is well known. We transfer it to symplectic spinor framework. We consider a symplectic manifold equipped with a metaplectic (i.e., symplectic spin) structure and a torsion-free symplectic connection. Vanishing of the (symplectic) Weyl curvature of the connection, gives rise to families of complexes of differential operators and consequently, to cohomology groups of these complexes. We describe properties of the cohomology groups that concern the primitive forms in the symplectic Weyl-flat case.

**Tim Lüders**: *Graded-equivariant bundle gerbes and twisted K-theory*

**Marián Lukáč**: *Courant Algebroids*
In this presentation we shall focus on the theory of Courant algebroids, that appears in various contexts in physics, for instance the geometry of (bosonic) Supergravity. We will explore mathematical properties of this theory as well as its interaction with the graded symplectic geometry. Next, we look at the role of vector bundle connections in this theory and illuminate all these topics with various examples.

**Antonella Marchesiello**: *Superintegrable families of magnetic monopoles in curved background*
We review some known results on the superintegrability of monopole systems in the three-dimensional (3D) Euclidean space and in the 3D generalized Taub-NUT spaces. We show that these results can be extended to certain curved backgrounds that, for suitable choice of the domain of the coordinates, can be related via conformal transformations to systems in Taub-NUT spaces. These include the multifold Kepler systems as special cases. The curvature of the space is not constant and depends on a rational parameter that is also related to the order of the integrals. New results on minimal superintegrability when the electrostatic potential depends on both radial and angular variables are also presented.

**Filip Moučka**: *Cn-generalized complex geometry*
The vector bundle $TM + T^*M$ comes equipped with a canonical symmetric pairing, which is a fundamental object when introducing standard generalized geometry. However, there is also an equally canonical skew-symmetric pairing, which motivates the introduction of Cn-generalized geometry. In this talk, I will discuss the basic features of Cn-generalized geometry with a special focus on the Cn-analogue of generalized complex structures.

**Jiří Nárožný**: *One-jets of groupoids – bisection construction approach*
In this talk I'll review basic findings of possible generalisation of bisection groups associated to strict higher Lie groupoids and discuss the relationship of their one-jets with homotopy

Lie-Rinehart pairs of these groupoids. It is joint work in progress with Branislav Jurčo.

**Katharina Neusser**: *Cone structures and conic connections*
A cone structure on a manifold $M$ is given by a closed submanifold $\mathcal{C} \subset \mathbb{P}TM$ of the projectived tangent bundle of $M$, which is submersive over $M$. Such geometric structures arise naturally in differential and algebraic geometry and they come often equipped with a conic connection, which specifies a distinguished family of curves on $M$ in directions of $\mathcal{C}$. In a joint work with Jun-Muk Hwang we defined an important local invariant for so-called characteristic conic connections, namely the cubic torsion. In this talk I will give a geometric interpretation of the cubic torsion. This talk is based on joint work in progress with Andreas Čap.

**Reamonn Ó Buachalla**: *Quantum exterior algebras and torsion free bimodule connections*
In the differential calculus approach to noncommutative geometry one normally starts with a first-order differential calculus (an algebraic structure modeled on the space of one-forms of a smooth manifold) and then extends to a differential calculus (an algebraic structure modeled on the de Rham complex of a smooth manifold). In the classical setting, this extension is simply given by the exterior algebra of the one-forms. However, in the noncommutative setting this is a much more subtle question. I will present a surprisingly simple presentation of the prolongation of a first-order differential calculus in terms of the bimodule map of a torsion-free bimodule connection. The Drinfeld-Jimbo quantum flag manifolds are taken as motivating examples.

**Pavle Pandzic**: *$SL(2)$ theory and quantum computing*
We consider the action of $G = GL_2(\mathbb{C}) \times GL_2(\mathbb{C}) \times GL_2(\mathbb{C})$ on the space $\mathcal{P}(V)$ of polynomials on $V = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$, where $\mathbb{C}^2$ is the standard representation of $GL_2(\mathbb{C})$. We describe explicit highest weight vectors, as well as the multiplicities, of all irreducible submodules of $\mathcal{P}(V)$. We apply this to obtain a description of the orbit decomposition of $V$ under the action of $G$, as well as under the action of the subgroup $K = U(2) \times U(2) \times U(2)$ of $G$. The latter gives a classification of the three-cubit states in quantum computing.
This is joint work with Jing-Song Huang and Soo Teck Lee.

**Sylvie Paycha**: *A prolegomenon to renormalisation in three movements: or a (desperate?) attempt to make the infinite finite*
This series of three lectures based on joint work with various coauthors, in particular with Li Guo (Rutgers Univ., Newark) and Bin Zhang (Sichuan Univ., Chengdu), aims to give a mathematical perspective on certain aspects of renormalisation. Broadly speaking, renormalisation, which has an intricate and fascinating history, comprises a set of techniques derived from quantum field theory, which are used to deal with infinities arising when calculating quantities by modifying their values to compensate for discrepancies. These lectures only provide a prolegomenon in that we do not claim to explain renormalisation in its full breath. In the language of perturbative quantum field theory, we are only dealing with a finite number of loops.
1. Exposition: from regularisation to renormalisation. We describe various regularisation techniques such as cut-off, dimensional, zeta and heat-kernel regularisation underlying renormalisation methods. We shall discuss how they can be used in number theory (the zeta function), quantum field theory (one loop Feynman integrals), microlocal analysis (traces of pseudodifferential operators) and index theory (super trace of a heat-operator).
2. Development: algebraic and analytic methods for renormalisation. Going from sums and integrals in one variable to sums and integrals in several variables requires renormalising when these diverge in order to deal with the so called "sub-divergences". We shall discuss how on the algebraic side, coproducts coupled with dimensional regularisation can come to the rescue. On

the analytic side, we shall show how meromorphic functions in several variables combined with analytic regularisation a la Speer can prove useful.
3. Recapitulation: how locality comes to the rescue. Applications. In the latter case of analytic renormalisation, we use the concept of locality as a leading thread. It is a mathematical counterpart of the well-known concept of locality underlying axiomatic quantum field theory. Accordingly, we need to enhance the categories of sets, algebras, coalgebras . . . to the locality framework. We shall describe how locality is used to renormalise meromorphic functions in several variables when evaluating them at poles.
One useful reference among many others: "Mathematical Reflections on Locality"L. Guo, S. Paycha, B. Zhang published in Jahresbericht der Deutschen Mathematiker-Vereinigung (2023).

**Tomáš Procházka**: *Harmonic oscillator, 2d free boson and integrability*
In this talk, I explore the connection between two-dimensional free boson theory and rational deformations of the one-dimensional quantum harmonic oscillator from the perspective of quantum integrability. Specifically, I demonstrate how the application of Darboux transformations to the harmonic oscillator generates an infinite family of exactly solvable quantum mechanical potentials, each corresponding uniquely to a state in the Fock space of the free boson theory. This approach offers a simple entry point into the ODE/IM correspondence, hinting at its broader applicability to other two-dimensional conformal field theories (joint work with Matěj Kudrna).

**Ľuboš Ravas**: *BRST and beyond*
In this presentation we are going to briefly introduce Becchi-Rouet-Stora-Tyutin formalism (BRST) with two examples: mechanical system with SO(3) symmetry and Yang-Mills (YM) theory. Next we will look at more general Batalin-Vilkovisky formalism (BV) - we add ghost/antighost antifields and examine the action integral, again on the example of YM theory. Finally we will define a homotopy equivalnece and special deformation retract (SDR) with the purpose of introducing homological perturbation lemma (HPL).

**Andrea Rivezzi**: *On the universal Drinfeld-Yetter algebra*
In "Uniqueness of Coxeter structures on Kac-Moody algebras", (Adv. Math. 347 (2019), 1–104), A. Appel and V. Toledano Laredo introduced a cochain complex of algebras that controls the existence of universal quantization functors of Lie bialgebras. This talk aims to present the structure of the first algebra of this cochain complex, namely the universal Drinfeld-Yetter algebra. Finally, we shall focus on its algebra structure, which is related to symmetric groups. This talk is based on arXiv:2404.16786.

**Martin Schnabl**: *On classical solutions in open string field theory*
I will present some new results concerning classical solutions in open string field theory.

**Urs Schreiber**: *Introduction to Hypothesis H*
The global (non-perturbative) definition of higher gauge fields requires choices of "flux-quantization laws". This is classical for electromagnetism (with Dirac quantization given by line bundles), and famous for RR-fields (whose quantization conjecturally is in topological K-theory). But when the higher Maxwell-type equations are non-linear in the fluxes (as they generically are in higher dimensional supergravity) then such Whitehead-generalized cohomology theories fail as flux-quantization laws, and what is needed are more general *non-abelian cohomology* theories (as in: non-abelian Poincaré duality).
I will first discuss the algebraic topology behind this statement, notably the generalization of the Chern character map from K-theory to nonabelian cohomology via classical methods of

dg-algebraic rational homotopy theory, following our monograph doi.org/10.1142/13422 .
Then I will turn to the example of 11d super-gravity, and explain how one viable flux quantization here (aka: a "model for the C-field") is given by twisted forms of Pontrjagin-Borsuk-Spanier's "Co-Homotopy"theory, and how the choice of this law ("Hypothesis H") provably implies phenomena that have been conjectured to hold in the completion of 11d super-gravity by "M-theory"– such as the half-integral (lambda/2-shifted) quantization of the 4-flux density.
This is reporting on joint work with Hisham Sati.
Some form of lecture notes will be available at: ncatlab.org/schreiber/show/Introduction+to+Hypothesis+H

**Libor Šnobl**: *On the incompleteness of the classification of quadratically integrable Hamiltonian systems in the three-dimensional Euclidean space*
In their seminal paper in Nuovo Cimento A, 52 (1967), 1061–1084, Makarov, Smorodinsky, Valiev and Winternitz presented a list of quadratically integrable natural Hamiltonian systems in the three–dimensional Euclidean space and identified them with systems separable in ortogonal coordinate systems. This result is one of the standard references in the theory of integrable and superintegrable systems and lead to numerous further developments. It was widely accepted as proof of the equivalence of quadratic integrability and separability in Euclidean 3D space. However, it became forgotten that the derivation of the list by Makarov et al. was based on one technical assumption which limits the universality of the above mentioned equivalence. We shall demonstrate that without that technical assumption a quadratically integrable however nonseparable 3D natural Hamiltonian system exists and the original derivation needs to be revisited.

**Rudolf Šmolka**: *Elements of Graded Lie Theory*
In this talk, we will present the definition of a graded Lie group as a group object in the category of $\mathbb{Z}$-graded manifolds. We will discuss some basic properties and associated concepts such as an action of a graded Lie group on a graded manifold or graded fundamental vector fields. Throughout the talk we will be accompanied by the Z-graded generalization of a general linear group, on which our ideas will be illustrated.

**Matouš Trnka**: *Projective invariants of non-torsal ruled surfaces*
There are several ways of describing a non-torsal ruled surface by differential invariants being preserved by projective transformations $\mathbb{RP}^3 \to \mathbb{RP}^3$. We will show some of the classical ones being constructed differently, including rather analytic or geometric viewpoint. Additionally, we will try to compute some specific quantities of an explicit examples of non-torsal ruled surfaces.

**Dominik Trnka**: *Unstraightening of an operad*
I describe operadic Grothendieck construction which encodes a symmetric categorical operad as bicategory with some extra structure. I will use examples such as trees and leveled trees. A special case is unstraightening of a monoidal category.

**Fridrich Valach**: *On full supergravity*
We show that generalised geometry provides a natural organising principle which leads to a surprisingly elegant and concise description of $N = 1$ supergravity in 10 dimension to all orders in fermions. This in particular confirms that generalised geometry provides a natural set of variables for studying the massless level of string theory. a byproduct of this formulation we obtain for free the compatibility with Poisson-Lie T-duality and the possibility of performing interesting "limits/twists"of the theory.

**Bruno Vallette**: *An operadic approach to deformation theory*
The fundamental principle of deformation problems states that they are faithfully encoded by differential graded (dg) Lie algebras in characteristic zero [Pridham, Lurie]. The universal formulas can be quite involved (Baker—Campbell—Hausdorff, gauge group action). It turns out that some large classes of examples are structured by stronger algebraic structures: dg associative algebras (morphisms of associative algebras), dg pre-Lie algebras (morphisms of operads), dg Lie-graph algebras (morphisms of properads). In these cases, the general formulas greatly simply and allows one to effectively perform computations. In this course, we will review this approach including a detailed treatment of the last abovementioned example which is new. Then, we will treat more relaxed algebraic structures encoding deformation problems: homotopy Lie algebras (infini-morphisms of algebras over an operad) and partition Lie algebras (any characteristic).

**Vera Vertesi**: *Giroux correspondence via convex surfaces*
TBA

**Vladimír Souček**: *The Jakobsen diagrams and unitary highest weight modules*

**Petr Vlachopulos**: *Advancements in the Yau's conjecture on the first eigenvalue*
In this poster we would like to present a progres within the Yau's conjecure on the first eigenvalue of the minimal hypersurface $M^n$ in the unit $(n+1)$-dimensional sphere. By combining the Bochner and Reilly technique, with the help of suitably defined boundary value problems, we will try to get closer to an optimal lower bound for the smallest eigenvalue of the Laplacian on $M^n$, which should be $n$. The aim of our modest approach is to lead us to an improvement of the result obtained by Choi and Wang, where the first eigenvalue is bounded from below by n/2 and then build upon it.

**Jan Vysoký**: *Graded Principal Bundles and Connections*
Principal bundles are fundamental for a geometrical description of symmetries. In physics, they provide a mathematical framework for gauge theories - the most succesful model of particle physics.
It is thus only natural to generalize this notion to the category of graded manifolds. This poses certain challenges, in particular to avoid some point-set definitions of ordinary differential geometry. We describe the notion of a graded principal bundle, graded Lie groups and their representations, and a graded analogue of connections. Graded frame bundle is presented as a prominent example.

**Thomas Weber**: *The crossed product calculus*
In noncommutative geometry the smash product is a generalization the of semidirect product, where groups are replaced by Hopf algebras. Allowing the presence of an additional 2-cocycle leads to the notion of crossed product algebra. In Hopf—Galois theory, such crossed product algebras correspond to cleft extensions. We give sufficient conditions for noncommutative differential calculi on the structure Hopf algebra and coinvariant subalgebra to extend to the crossed product algebra. It is further shown that the noncommutative Atiyah sequence is exact in this framework. Moreover, the crossed product calculus is complete in the sense of Durdevic and we discuss the emerging braiding, together with examples. This talk is based on joint work with Antonio Del Donno, Emanuele Latini and Andrea Sciandra.

**Henrik Winther**: *Higher order connections in noncommutative geometry*
We prove that a system of higher order connections is equivalent to a notion of phase space quantization, in the setting of noncommutative differential geometry. Further we show that higher

order connections are equivalent to (ordinary) connections on jet modules. This involves introducing the notion of natural linear differential operator, as well as an important family of examples, namely the Spencer operators, generalizing their corresponding classical analogues. These Spencer operators form the building blocks of this theory by providing conversions between the different representations of higher order connections as left/right splittings of the jet exact sequences and as explicit pieces of the necessary jet connections which are constructed using the data of a left connection on the first-order differential calculus. A system of such higher order connections then gives a quantization in the sense of a splitting of the projection from differential operator to their symbols. This yields total symbols and star products, i.e. phase space quantizations, but where we allow the position coordinates to form a possibly noncommutative algebra. Joint work with K. Flood and M. Mantegazza.

**Lenka Zalabová**: *Conserved quantities for conformal loxodromes on conformal sphere*
We study conformal circles and conformal loxodromes on conformal spheres. We combine tractor calculus and the fact that the sphere is homogeneous to find some conserved quantities for loxodromes. This is a joint work in progress with Prim Plansangkate.

**Martin Zika**: *Spans of Quantum L-infinity Algebras*
Quantum $L_\infty$ algebras are loop homotopy generalizations of Lie algebras which encode quantum gauge theories in the Batalin-Vilkovisky formalism. We will show that any two quasi-isomorphic theories can be connected by a span of formal non-linear morphisms; we will construct a "master theory"which projects down to both theories. We will compare this construction with the quantum odd symplectic category of (generalized) Lagrangian relations and half-densities.

**Alexander Zuevsky**: *Applications of CFT determinant formulas in number theory*
In this talk we show how to use the determinantal representations for correlation functions in CFT to derive new determinant formulas for powers of the modular discriminant expressed via deformed elliptic functions with parameters in number theory.

# GENERAL INFORMATION

45rd WINTER SCHOOL GEOMETRY AND PHYSICS
Srní, Czech Republic
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https://conference.math.muni.cz/srni/

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