Asymptotically Kerr-de Sitter Spacetimes: Necessary Conditions

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Solution: Conformal Killing–Yano 2-forms

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Study obstructions asymptotically.

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We can study asymptotically Kerr-NUT-dS spacetimes by studying "conformally-invariant" series expansion of $\mathcal{Q}^{\Omega^2 g}$ *near* $\partial\bar{M}$ *in* Ω.

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$$0 \overset{\partial \bar{M}}{=} \top(\nabla_n - H) Q_{ab} + \bar{\nabla}_{[a} Q^{\top}_{nb]}$$

$$0 \overset{\partial \bar{M}}{=} (d-2) n^b \bar{g}^c_a (\nabla_n Q)_{bc} - (d-2) H Q^{\top}_{na} + \bar{\nabla}^b Q^{\top}_{ab}$$

$$0 \overset{\partial \bar{M}}{=} \overline{CKY}(Q^{\top})_{abc}$$

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and "normal vector" like section $[\rho := -\frac{1}{d}(\Delta\Omega + J\Omega)]$

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Conformally-invariant "normal derivative":

$$I \cdot \hat{D} := (\nabla_n + w\rho) - \frac{\Omega}{d+2w-2}(\Delta + wJ) \,.$$

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\mathring{F}^c{}_{(a} Q^\top_{b)\circ c} &\overset{\partial\bar{M}}{=} 0 \\
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2 evolution- + 2 boundary constraint-type equations. This is the expected general trend.

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Thank you!