The B model field theory in BV formalism – Eugenia Boffo

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joint work in progress w/ Hulik, Sachs

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Plan of the talk:

1. B model
1(i). its field theory

2. Batalin-Vilkovisky

3. classical BV action

1. B model

topological sigma model — $\frac{\delta}{\delta h}\langle \mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)\rangle = 0$ — h metric

has an AKSZ formulation — $\mathrm{Map}(T[1]\Sigma,\ T[1]T^*M)$, d, $\omega,\ \mathcal{H}\ (X_{\mathcal{H}})$

topological twist of (2,2) SCFT — $\{G,G\}\sim T$, $\{G',G'\}\sim T'$ $\longrightarrow$ $G\pm G' =: G^{\pm}$, $\{G^+,G^+\}=0$

mirror symmetry: A model

1(i). Effective background field theory

moduli space of the B model : complex structures

the field theory is constructed at a point in moduli space

$$\mathfrak{F}\subset \Omega^{0,\bullet}(M,\Lambda^\bullet TM_{(1,0)}) \equiv PV^{\bullet,\bullet}$$

$\mu\in PV^{1,1}$ — $\bar\partial\mu + \frac{1}{2}[\mu,\mu]=0 \longleftrightarrow$ integrability of almost complex str.

$\rho\in PV^{0,0}$ — $\partial_\Omega\mu + \bar\partial\rho + [\mu,\rho]=0 \longleftrightarrow$ preservation of the holo. volume form

$\partial_\Omega := \Omega^{-1}\circ\partial\circ\Omega$, $\partial_\Omega : PV^{\bullet,\bullet}\to PV^{\bullet-1,\bullet}$, $\partial_\Omega^2=0$

2. Batalin-Vilkovisky

structure — $(T^*[-1]\mathfrak{F},\ \omega,\ Q)$ with Φ^*,Φ, w/ $\mathcal{L}_Q\omega=0$ $\Longrightarrow \exists\, S,\ \underbrace{\{S,S\}=0}_{\text{CME}}$

solution to the CME — $S = S[\Phi] + \Phi^* Q\Phi$

quantum: Δ BV Laplacian, — $\Delta\, e^{\frac{i}{\hbar}S}=0$

BV complex $\overset{1:1}{\leftrightarrow}$ to L_∞ algebra — $\{m_i : V^{\otimes i}\to V\ +\ \text{conditions}\}$

3. BCOV (Kodaira-Spencer) in BV formulation

Bershadsky - Cecotti - Ooguri - Vafa '93

state of the art:

- M CY_3, $\mu' \equiv \mu\vdash\Omega\in\Omega^{2,1}$, $\mu'\in \mathrm{im}(\partial)$
  "Kinetic term" $\int_M \mu'\,\bar\partial\,\frac{1}{\partial}\,\mu'$
- Costello – Li '19 : BV - interaction term

observation: Tian lemma : $[\mu,\mu] = \partial_\Omega(\mu\wedge\mu)$

$\rightsquigarrow$ if $\tilde{A}\in PV^{2,1}$ the MC can be written as $\begin{cases}\bar\partial(\mu+\partial_\Omega\tilde{A})=0\\ \bar\partial\tilde{A}+\mu\wedge\mu=0\end{cases}$

Model:

(B-Hulík-Sachs '25?) WIP

$\mathfrak{F}\subset PV^{\bullet,\bullet}[2]((u))$,

ghost degree: if $A\in PV^{k,m}[2]((u))$, $|A| = 2-(k+m+2\,\mathrm{pow}(u))$

$$a = \overset{(1,1)}{\mu} + u\overset{(0,0)}{\rho} + \frac{1}{u}\overset{(2,1)}{\tilde{A}}$$

$Q = \bar\partial + \underbrace{u\partial_\Omega}_{=:\Upsilon}$, $\quad m_2(A,B) = u^{\#}A\wedge B$

$(A,B) = \delta_{|A|,-|B|}\int_{M_3}\oint du\ \Omega\wedge(A\wedge B)\vdash\Omega \neq 0$ iff the argument $\in\Omega^{3,3}$

$m_2,\ Q,\ \Upsilon,\ (\cdot,\cdot)$ compatible (richer) A_2 structure

$$S = (a,\ Q(a+\Upsilon a) + m_2(a,a)) + (a^*,\ Q(c+\Upsilon c) + m_2(a,c)) + (c^*,\ m_2(c,c))$$

with $c = pv^{1,0} + \frac{1}{u}pv^{2,0}$

$a^* = pv^{1,2} + u\,pv^{0,1} + \frac{1}{u}pv^{2,2}$

$c^* = pv^{1,1} + u\,pv^{0,0} + \frac{1}{u}pv^{2,1}$