Uniqueness and multiplicity of large positive solutions

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Abstract: Establishing the uniqueness of the large positive solution of the semilinear equation
\[ \Delta u = a(x)f(u) \]
on a bounded domain, \( \Omega \), of \( \mathbb{R}^N \), \( N \geq 1 \), is imperative from the point of view of the applications of the theory of differential equations in Population Dynamics, \[3\]. When \( a \equiv 1 \), the existence of positive large solutions for (1) was established by J. B. Keller \[1\] and R. Osserman \[10\] in 1957. Essentially, in such case (1) has a large solution if and only if \( f(u) \) satisfies the Keller-Osserman condition, which entails the existence of universal a priori bounds for the positive solutions of (1). Although there is a substantial amount of literature establishing the uniqueness of the large positive solution of (1) if \( f(u) \) is increasing, among them, those of M. Marcus and L. Véron \[9\] and J. López-Gómez, L. Maire and M. Véron \[7\], as well as those of \[2\] and \[4\], even the precise role of the Keller–Osserman condition is far from well understood yet, \[5\].

This talk discusses some very recent advances on the problem of the uniqueness and multiplicity of the large positive solutions of (1). In particular, by adopting a dynamical perspective, constructs multiple large solutions of (1) when \( f(u) \) is increasing, except on an arbitrarily small neighborhood of finitely many (prescribed) points where it exhibits a pulse-type behavior, and shows the uniqueness of the large solution if \( f(u) \) is sufficiently close to an increasing function. Moreover, it discusses the recent multidimensional sharp uniqueness theorems of \[7\] and the astonishing one-dimensional multiplicity results of \[8\].

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References